CSE 421 - Lecture 22

Wednesday, February 28, 2018 2:26 PM

s-t cut

$$\text{cap}(A,B) = \sum_{e \text{ out of } A} c(e)$$

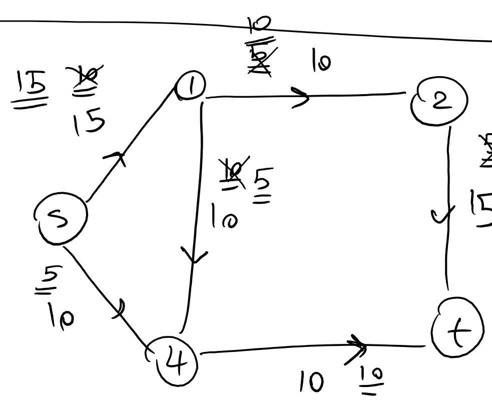
$f \to$ capacity $f(e) \le c(e) \quad \forall e$

$\quad \to$ conser $\sum_{e \text{ out of } v} f(e) = \sum_{e \text{ into } v} f(e) \quad \forall v \ne s,t$

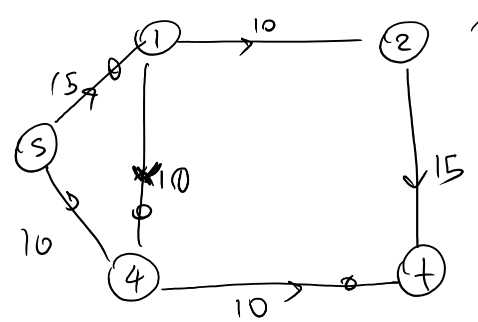
$$v(f) = \sum_{e \text{ out of } s} f(e).$$

Cl 1: $\forall f, \ \forall A,B \qquad v(f) = \sum_{e \text{ out of } A} f(e) - \sum_{e \text{ into } A} f(e)$

Cl 2: $\forall f, \ \forall A,B \qquad v(f) \le \text{cap}(A,B).$



G_f



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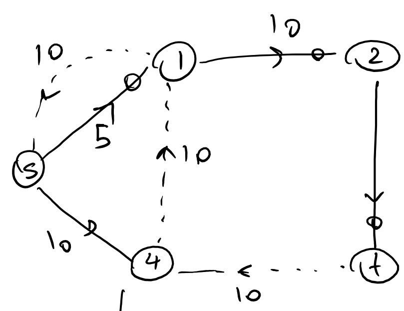
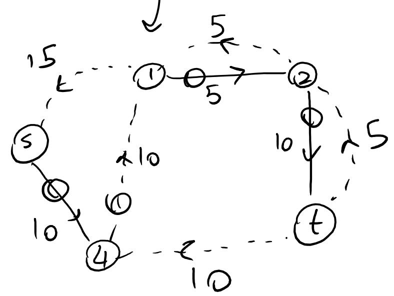
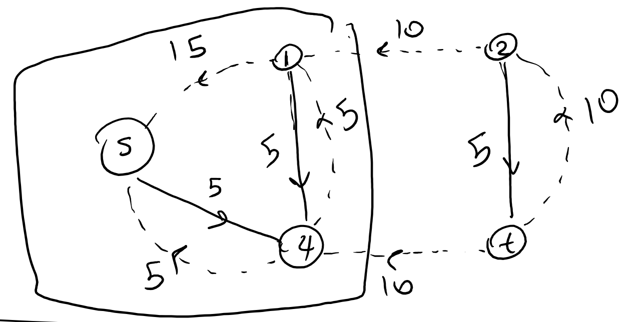


$c_f(e) = c(e) - f(e)$

$c_f(e^R) = f(e)$





Suppose no path in G^f exists

Then $\exists$ cut A,B s.t. $\text{cap}(A,B) = v(f)$.

A = all vertices v that there is a path $s \rightsquigarrow v$ in G^f.

A = all vertices v, that there is a path $s \rightsquigarrow v$ in G_f.

B = everything else.

- $s \in A$ ✓

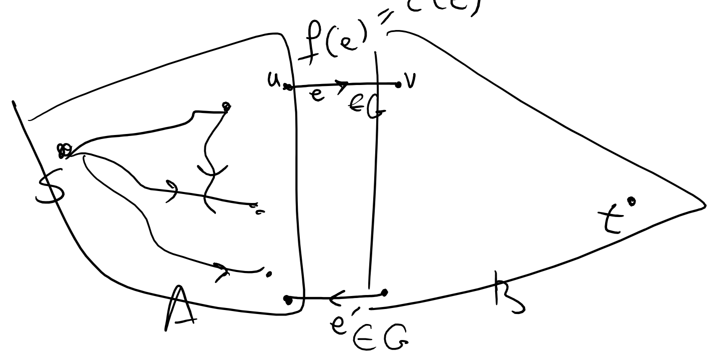
- $t \notin A$

$$v(f) = \sum_{e \text{ out of } A} f(e) - \sum_{e \text{ into } A} f(e)$$

$$= \sum_{e \text{ out of } A} c(e)$$

$$= cap(A, B).$$

G_f



$e \notin G_f \Rightarrow c_f(e) = 0 \Rightarrow c(e) = f(e)$

$e'^R \notin G_f \Rightarrow c_f(e'^R) = 0 \Rightarrow f(e') = 0.$