

Claim: S^* is man optimal.

Pf. By contradiction?

$\Rightarrow \exists$ man not getting his best valid partner

$\Rightarrow \exists$ man rejected by his best option.

A man Y is first rejected by best option A

Say A 's current match in S^* is Z

$\Rightarrow \exists$ stable matching S that $(A, Y) \in S$

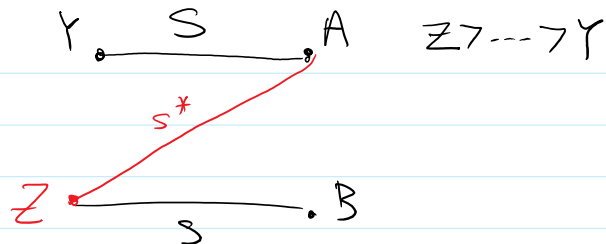
If Z prefers A to B then (A, Z) is unstable for S and we get a contradiction.

$\Rightarrow Z$ prefers B to A .

$\Rightarrow Z$ is rejected by B .

Z also rejected from his best option!

Contradiction!



$B \succ \dots \succ A$

Women get their worst valid partner

By contradiction.

Say $(A, Z) \in S^*$ but Z is not A 's worst option

Say Y is worst valid partner of A

$\exists S$ (stable matching) that $(A, Y) \in S$.

Z prefers A to B because GS ALG is man-optimal

(A, Z) is unstable for S
contradiction!

