Set $S$ is all vertices to which we have found shortest paths.

$$\min_{u \in S} d[u] + c_{u,v} = d[v]$$

If $v \not\in S$ then $d[v]$ is our best guess of shortest path.

Pick $v$ min $d[v]$ over all vertices not in $S$.

Add to $S$.

Update dist

Recurse

Correctness by induction

At any point of running ALG, for all $u \in S$, $d[u]$ is the shortest path from $s \rightarrow u$.

Base case: $|S| = 1$, $S = \{s\}$, $d[S] = 0$.

IH: Suppose claim holds when $|S| = k$.

IS. Goal: show claim holds when we go to $|S| = k+1$.

Suppose we added $v$.

$$v = \min_{w \not\in S} d[w]$$

$$d[v] = \min_{w \not\in S} d[w] + c_{u,v}.$$ for $u \in S$

Goal: $P_v$ is shortest path to $v$.

Show $\text{len}(P) \geq \text{len}(P_v)$.

$\text{len}(P) \geq \text{len}(s \rightarrow x \text{ path}) + c_{xy} \geq d[x] + c_{xy} \geq d[y] \geq d[v] = \text{len}(P_v)$.

By IH