CSE421: Design and Analysis of Algorithms	March 1st, 2018
Homework 8	
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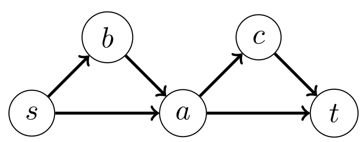
Please see https://courses.cs.washington.edu/courses/cse421/18wi/grading.html for general guidelines about Homework problems.

Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers.

P1) Let G be a weighted directed graph where every edge has a capacity $c[e]$. We have stored water at a number of nodes in this graph. Suppose that node i has a reserve of $r[i]$ (liters) of water; note that $r[i]$ can be zero for many nodes in this graph. On the other hand, each node has a demand of water. Node i need $d[i]$ (liters of) water. You can assume that $\sum_i r[i] \geq \sum_i d[i]$. Design an algorithm that runs in time polynomial in $n, \max_e c[e]$ and outputs if there exists a way to send water from the storages to the nodes and satisfy all demands while sending at most $c[e]$ amoung of water on every edge. Your algorithm just needs to output yes/no.

P2) Given a directed graph $G = (V, E)$, a pair of vertices s, t and an integer k. We want to output yes if there are k *vertex* disjoint paths from s to t and no otherwise. For example, in the following graph there are two edge disjoint paths from s to t but no two vertex disjoint paths from s to t. Design a polynomial time algorithm for this problem.



P3) Give a polynomial time algorithm to find the minimum vertex cover in a bipartite graph.

a) Construct a flow network from the input bipartite graph just as in the maximum matching algorithm.

b) Show that every min-cut in this flow network gives a vertex cover whose size is the same as the capacity of the cut.

c) Show that every minimum sized vertex cover in the bipartite graph gives a cut whose capacity is the same as the size of the vertex cover.

d) Write down the algorithm and prove that it works.

P4) 3-Color problem is defined as follows: Given a graph $G = (V, E)$, does it have a 3-coloring?

4-Color problem is defined as follows: Given a graph $G = (V, E)$, does it have a 4-coloring?

Prove that 3-Color $\leq_P$ 4-Color.

P5) **Extra Credit:** Let $G = (V, E)$ be a directed graph such that the indegree of every vertex is equal to its outdegree and that is equal to k. Let $T \subseteq E$ be a spanning tree of G; this means that if we drop the direction of the edges of G, T, T will be a spanning tree of the underlying graph. Let $0 < \alpha < 1$ be fixed parameter such that T satisfies the following inequality for every cut $(S, V-S)$ of G:

$$|T(S, V-S)| \leq \alpha |E(S, V-S)|$$

where the $|T(S, V-S)|$ is the number of edges of T in the cut $(S, V-S)$ and the $|E(S, V-S)|$ is the number of edges of G in the cut $(S, V-S)$.

Prove that G has a TSP tour of length at most $O(nk\alpha)$, i.e., a sequence of vertices $v_1, v_2, \ldots, v_r$ for $r = O(nk\alpha)$ such that $v_1 = v_r$, each vertex appears at least once in this sequence and that for all i, $(v_i, v_{i+1}) \in E$