**CSE421: Design and Analysis of Algorithms** January 18, 2018

# Homework 3

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Please see https://courses.cs.washington.edu/courses/cse421/18wi/grading.html for general guidelines about Homework problems.

Most of the problems only require one or two key ideas for their solution. It will help you a lot to spell out these main ideas so that you can get most of the credit for a problem even if you err on the finer details. Please justify all answers.

P1) We say that the distance between two nodes $u, v$ in a graph $G = (V, E)$ is the minimum number of edges in a path joining them; we'll denote this by $dist(u, v)$. We say that the *diameter* of $G$ is the maximum distance between any pair of nodes. Design an algorithm that runs in time $O(n(n + m))$ and finds the diameter of $G$, i.e., it outputs two vertices $u, v$ that has the maximum distance.

P2) Given an undirected graph $G = (V, E)$, design an $O(m + n)$ time algorithm to detect wether $G$ has a cycle. If $G$ has a cycle, your algorithm should output the cycle.

P3) Given a connected undirected graph $G = (V, E)$, design an $O(m+n)$-time algorithm to find a vertex in $G$ whose removal *does not* disconnect $G$. Note that as a consequence this algorithm shows that every connected graph contains such a vertex.

P4) Given a graph $G = (V, E)$ such that the degree of every vertex of $G$ is at most $k$. Show that we can color the vertices of $G$ with $k + 1$ colors such that the endpoints of every edge of $G$ have distinct colors. For example, if $k = 1$ then the graph is always bipartite.

P5) **Extra Credit:** Prove that we can color the edges of every graph $G$ with two colors (red and blue) such that, for every vertex $v$, the number of red edges touching $v$ and the number of blue edges touch $v$ differ by at most 2.