

Thm Correctness of Dijkstra

Proof by induction.

" S is the set of vertices we find the shortest path from s "

$P(k)$ = "at iter k , for all $u \in S$, $d[u]$ is the shortest path distance from s to u "

Base: $k=0$. $S = \{s\}$. $d[s] = 0$. \checkmark

IH: Suppose $P(k)$ holds.

$$\text{len}(P) \geq \text{len}(P_v)$$

IS: Suppose we add v into S .

$$d[v] = d[u] + C_{u,v}$$

Goal: P_v is the shortest path $s \rightarrow v$

$$\text{ie } \text{len}(P) \geq \text{len}(P_v)$$

Let y be the first point in P outside S , x be the point before y .

$$\text{len}(P) \geq \text{len}(s \rightarrow x) + C_{x,y}$$

$$(IH) = d[x] + C_{x,y}$$

$$(\text{choice of } v) \geq d[u] + C_{u,v}$$

$$= d[v] = \text{len}(P_v)$$

(choose $v \notin S$ st
it min
 $d[u] + C_{u,v}$)

So, we know the shortest path from s to v .



