Reduced schedule: schedule that insert item only when requested.

**Lemma:** We can transform schedule $S$ to a reduced schedule $S'$ without increasing the number of misses.

**Proof:** by induction on the number of unreduced misses.

Idea: want to decrease the number of unreduced misses by 1.

Suppose $S'$ brings $d$ into cache at time $t$ without request.

Let $c$ be the item $S$ evicts.

**Case 1:** $d$ is requested later before it gets evicted.

just evict $c$ when it is used.

**Case 2:** $d$ is never requested before evicted.

$S' \leq$ schedule has $\alpha$ many unnecessary moves

$S' \leq$ schedule has $\alpha - 1$
Thus FIF is optimal.

Proof by induction. (exchange argument)

Let $S_e$ be the schedule by FIF.

$P(j) =$ "There is an optimal schedule $S \leq S_e$ for the first $j$ steps."

**base case:** Lemma

**IH:** $P(j)$ is true

**IS:** Let $S$ be the schedule given by $P(j)$.

(assumed, $S = S_e$ for first $j$ steps and $S$ is reduced and optimal)

Let $d$ be the $(j+1)$th request.

**Case 1:** $d$ is in the cache

$S' = S$. $S'$ satisfies $P(j+1)$

**Case 2:** $d$ is not the cache

and ($S$ and $S_e$ evicts the same item).

$S' = S$. done.

**Case 3:** $d \in S$, $S_e$ evicts $e$, $S$ evicts $f \neq e$. 
Case 3: \( d \in S \), \( S \) evicts \( e \), \( S \) evicts \( f \neq e \).

Step j

\[ \begin{align*}
&\text{same} & \text{elf} & \text{same} & \text{elf} \\
&\text{same} & \text{elf} & \text{same} & \text{elf}
\end{align*} \]

Step j+1

\[ \begin{align*}
&\text{same} & \text{elf} & \text{same} & \text{elf} \\
&\text{same} & \text{elf} & \text{same} & \text{elf}
\end{align*} \]

Let \( j' \) is the first time \( S \) and \( S' \) take different actions.

Let \( y \) be the item requested.

Case 3a \( y = e \)

Case 3b \( y = f \)