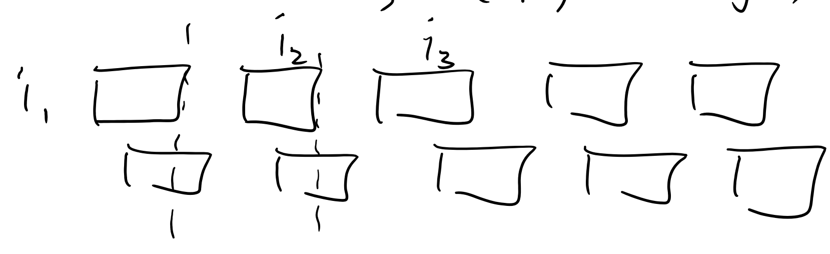
Thm Greedy (earliest finish time) is optimal ~~for interval scheduling~~

Proof Let $i_1, i_2, \ldots, i_k$ be jobs greedy picked

Let $j_1, \ldots, j_m$ be any valid sol'n. (think it as OPT)

Claim: For all r, $f(i_r) \le f(j_r)$

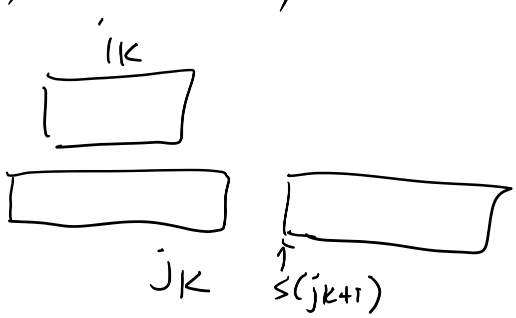


Proof by induction

base case $f(i_1) \le f(j_1)$ (Greedy picks the job finish first)

IH: $f(i_k) \le f(j_k)$

IS:



$$f(i_k) \le f(j_k) \le s(j_{k+1})$$

So, j_{k+1} is one of candidate greedy consider.

Since greedy picks min finish time.

$$f(i_{k+1}) \le f(j_{k+1})$$

---

why $k \ge m$?

why $k \geq m$?

Consider step $m-1$.

$$f(i_{m-1}) \leq f(j_{m-1})$$

so, job j_m is again candiate.

so, greedy must pick some job.

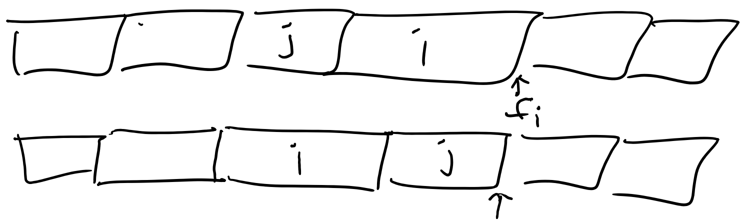
t6

<u>lemma</u> Swapping 2 adjacent inverted jobs does not $\uparrow$ max L.

Proof

before

after swapping



Note $L_i = \begin{cases} 0 & \text{if } f_i \leq d_i \\ f_i - d_i \end{cases}$

Let L_i' is the lateness after swapping

$(d_i < d_j)$

$$\begin{cases} \bullet\ L_k' = L_k \leq \max L \quad \forall\ k \notin \{i, j\} \\ \bullet\ L_i' \leq L_i \leq \max L \\ \bullet\ L_j' = f_j' - d_j = f_i - d_j < f_i - d_i = L_i \leq \max L \end{cases}$$

$\Rightarrow \max L' \leq \max L$

Lem 2 There is an optimal schedule st
jobs are sorted according deadline in ↑ order.

Pf bubble sort + lem 1.

Thm The greedy is optimal

Pf The schedule found in Lem 2 ≠ greedy

They only differ for jobs with same deadline.

but they doesn't affect max L

So greedy is optimal.
