- $\max c^T x$
  $Ax = b$
  $l_i \le x \le u_i$

$$\begin{array}{cc} & m \\ n & \boxed{A} \end{array}$$

Currently: $m^{\omega} \log(\frac{1}{\varepsilon})$

each col has 2 non-zero $\to$ $m\sqrt{n}$.

$m^4 \log(\frac{1}{\varepsilon})$ (next lecture)

$\text{OPT} = \min_{\substack{Ax=b \\ x \ge 0}} c^T x$, (min s-t cut : s-t flow $\le$ s-t cut)

Q: Can we find a lower bound of OPT.

A: say we have y st $A^T y \le c$

$$y^T b = y^T A x \underset{\uparrow}{\le} c^T x \quad \forall x \text{ st } \begin{array}{l} Ax = b \\ x \ge 0 \end{array}$$

used $x_i \ge 0$ $\forall i$

So, we know

$$y^T b \le \text{OPT}$$

<u>Lem</u>

$$\max_{\substack{A^T y + s = c \\ s \ge 0}} y^T b \le \min_{\substack{Ax = b \\ x \ge 0}} c^T x$$

(generalization of max flow $\le$ min cut)

<u>Thm</u> If either left or right has finite sol'n

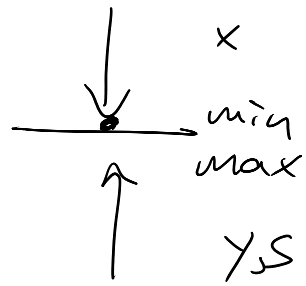
Thm If either left or right has feasible sol'n

Then $\max y^T b = \min c^T x$

$A^T y + s = c$ $Ax = b$

$s \geq 0$ $x \geq 0$

(dual)   (primal)



Obs: Given (x, s, y) st $Ax = b$, $A^T y + s = c$, $x, s \geq 0$

Then $c^T x - b^T y = (\cancel{A^T y} + s)^T x - \cancel{(Ax)^T} y$

$= x^T s$ $(x_i, s_i \geq 0)$

feasible (x, s, y) is sol'n $\Longleftrightarrow$ $x_i s_i = 0 \quad \forall i$

$\forall t$, def (x_t, s_t, y_t) st

$Ax_t = b$, $A^T y_t + s_t = c$, $x_t \geq 0$, $s_t \geq 0$

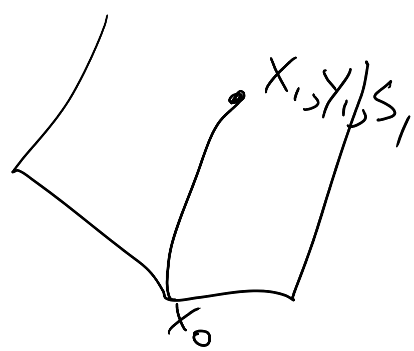
Interior Point Method

$(x_t)_i (s_t)_i = t \quad \forall i$

Thm x_t, s_t, y_t exists

- Q: how find a point

$Ax = b$, $x \geq 0$



We

$A\left(x - \lambda \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}\right) = b$, $x - \lambda \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \geq 0$