Indeped set:
Given $G = (V, E)$, $k \in \mathbb{Z}$.
Is there $S \subseteq U$ s.t. $|S| \geq k$ s.t. no 2 vertices in $S$ are joined by edge.

Clique:
Given $G = (V, E)$, $k$.
Is there $S \subseteq U$ s.t. $|S| \geq k$ s.t. every 2 vertices in $S$ are joined by edge.

Claim: Ind set $\leq P$ clique, clique $\leq P$ Ind

Proof: Let $x = (G, k)$

$s(x) = (\overline{G}, k')$ where $e' \in G \iff e \notin \overline{G}$

$S$ is indep in $G$ $\iff$ $S$ is clique in $\overline{G}$

Vertex Cover:
Given $G = (V, E)$, $k$.
Is there $S \subseteq U$ s.t. $|S| \leq k$ s.t. every edge has 1 end pt in $S$.

Claim: Vertex cover $\geq$ indep set.
Proof:
\[ X = (G, k) \quad (\text{indep set}) \]
\[ f(X) = (G, n-k) \]

Need to prove: If \( S \) is indep then \( V-S \) is vertex cover.

\[ \Rightarrow \]

If \( \forall e \in E \), only 1 end pt in \( S \) has \( \leq 1 \) end pt in \( V-S \).

Set Cover:

Given \( U \), collection of subsets \( S_1, \ldots, S_m \subseteq U \), \( k \).

Is there \( k \) sets such that \( \bigcup_{i=1}^{m} S_i \supseteq U \)?

Claim: Vertex cover \( \leq \) set cover

Proof:
\[ X = (G-(V,E), k) \quad (\text{vertex cover}) \]
\[ f(X) = \bigcup U = E \]
\[ S_v = \{(u,v) \in E \} \]

\[ \exists \rightarrow \exists \neg \exists R \rightarrow K \]
T is vertex cover \iff \{ S \cup \{ u \} \mid u \in T \} is set cover