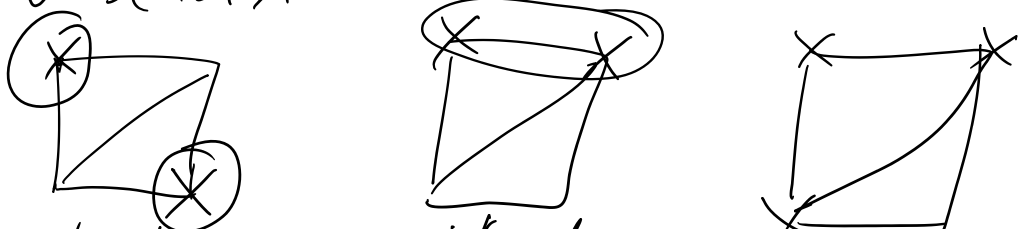
Lecture 25

Wednesday, November 28, 2018 1:06 PM

Independ set:

Given $G=(V,E)$, $k \in \mathbb{Z}$.

Is there $S \subseteq V$ st $|S| \geq k$ st no 2 vertices in S are joined by edge



✓ ind ✗ clique | ✗ ind ✓ clique | ✗ ind ✓ clique

Clique:

Given $G=(V,E)$, k.

Is there $S \subseteq V$ st $|S| \geq k$ st every 2 vertices in S are joined by edge

<u>Claim</u> Ind set $\leq_p$ clique, clique $\leq_p$ Ind

<u>Pf</u> Let $X = (G, k)$

$f(X) = (\overline{G}, k)$ $\quad e \in G \iff e \notin \overline{G}$

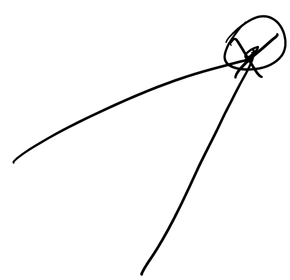
S is indep in G $\iff$ S is clique in $\overline{G}$

Vertex Cover:

Given $G=(V,E)$, k

is there $S \subseteq V$ st $|S| \leq k$ st

every edge has 1 end pt in S



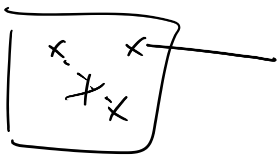
<u>Claim</u> Vertex Cover $\geq$ indep set

Claim Vertex Cover $\leq$ Indep set

Proof $\quad X = (G, k) \quad$ (indep set)

$f(X) = (G, n-k)$

Need to prove: S is indep iff $V-S$ is vertex cover

"$\Rightarrow$"  $\forall e \in E$, only 1 end pt in S

$\Rightarrow \forall e \in E$ has ≥ 1 end pt in $V-S$

"$\Leftarrow$" same

Set cover:

Given U, collection of subsets $S_1, \ldots, S_m \subseteq U$, K

Is there k sets st $\boxed{\bigcup_i S_i} \supseteq U$

$S_{i_1} \cup S_{i_2} \cdots \cup S_{i_k}$

Claim Vertex Cover $\leq$ set cover

Proof: $X = (G=(V,E), k) \quad$ (vertex cover)

$$f(X) = \begin{cases} U = E \\ S_v = \{(u,v) \in E\} \end{cases}$$

T is vertex cover $\Longleftrightarrow$ $\{S_v\}_{v \in T}$ is set cover