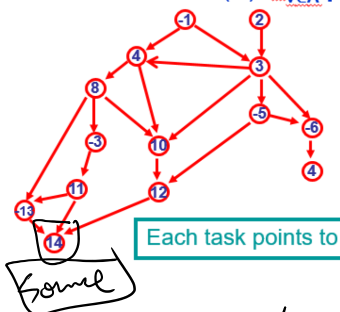
Lecture 24b

Monday, November 26, 2018 11:59 AM

Project Selection

- Given a DAG **G=(V,E)** representing
  precedence constraints on tasks
  (a task points to its **predecessors**)
  a profit value **p(v)** for each task **v∈V**
  (may be positive or negative)
- Find a set **A⊆V** of tasks that is closed under predecessors,
  (i.e. if **(u,v)∈E** and **u∈A** then **v∈A**)
  that maximizes Profit(**A**)=$\Sigma_{v\in A}$ **p(v)**



Each task points to its predecessor tasks

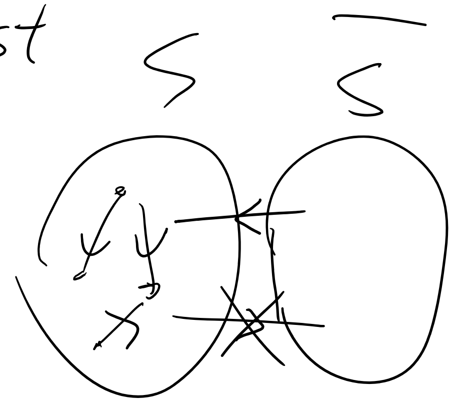
2

p(v) can be negative

Want: find a cut $(S,\bar{S})$ st

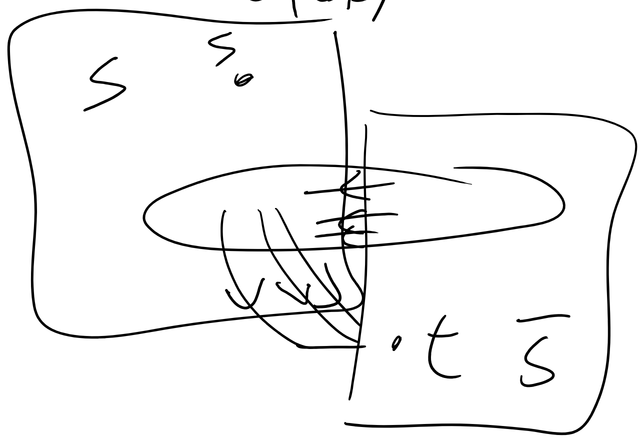
(1). $\sum_{v\in S} p(v)$ is max

(2). satisfies predecessor constraints

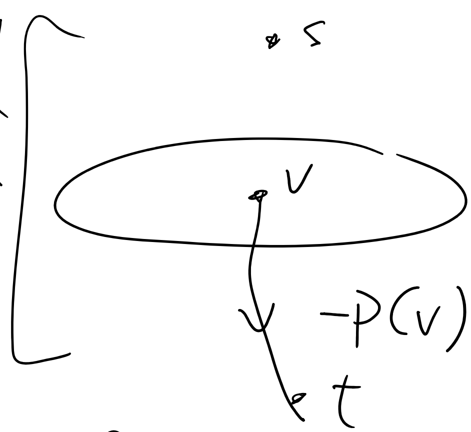


Obs: If every edge in G has cap $+\infty$,

then $cap(S,\bar{S}) < +\infty$ if and if (2) satisfied.

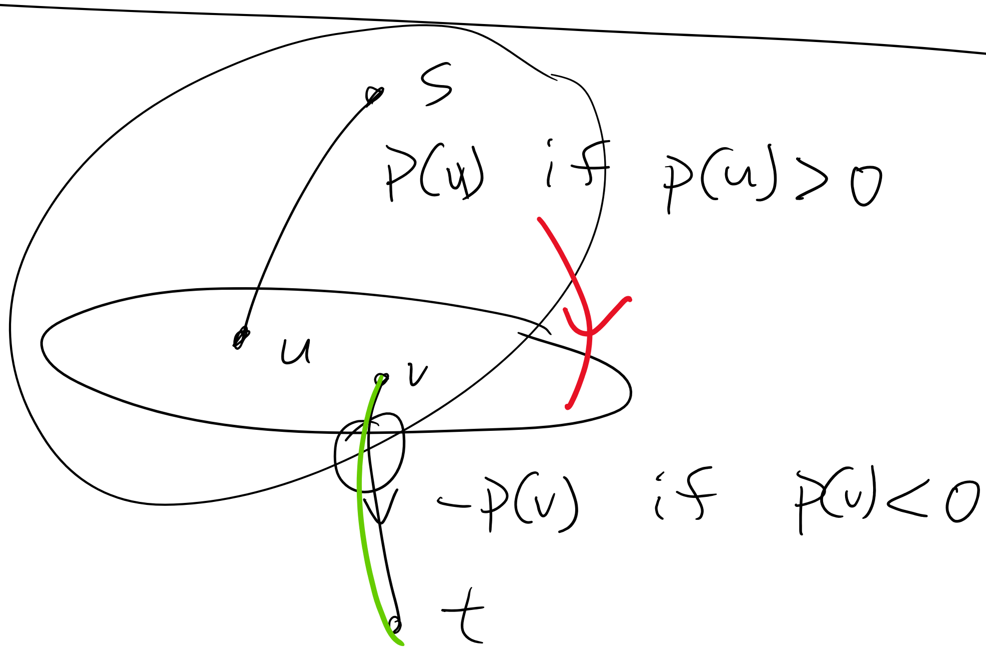


bad idea



$cap(S,\bar{S}) = -\sum_{v\in S} p(v) +$ ∞ if (2) violated, 0 else

$$\mathrm{Cap}(S,\bar{S}) = -\sum_{v\in S} p(v) + \quad 0 \text{ else}$$



$$\mathrm{Cap}(S,\bar{S}) = \sum_{\substack{v\in S\\ p(v)<0}} -p(v) + \sum_{\substack{v\in\bar{S}\\ p(v)>0}} p(v)$$

$$= -\sum_{\substack{v\in S\\ p(v)<0}} p(v) + \left(\sum_{p(v)>0} p(v) - \sum_{\substack{v\in S\\ p(v)>0}} p(v)\right)$$

$$= \underbrace{\sum_{p(v)>0} p(v)}_{\text{independent set } S} - \sum_{v\in S} p(v)$$

<u>Thm</u> Let $(S,\bar{S})$ be min s-t cut of G.

Then the $(S-\{s\})$ max profit