Baseball Elimination

<table>
<thead>
<tr>
<th>Team</th>
<th>Won-Lost</th>
<th>Left</th>
<th>NYY</th>
<th>BAL</th>
<th>BOS</th>
<th>TOR</th>
<th>DET</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York Yankees</td>
<td>75-59</td>
<td>28</td>
<td>3</td>
<td>8</td>
<td>7</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Baltimore Orioles</td>
<td>71-63</td>
<td>28</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Boston Red Sox</td>
<td>69-66</td>
<td>27</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Toronto Blue Jays</td>
<td>63-72</td>
<td>27</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Detroit Lions</td>
<td>49-86</td>
<td>27</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

49 + 27 = 76

Question: Can Detroit come to first place?

Assume:
- No game can tie
- Every game happens.

Notice: Detroit win ≤ 76 games.

=> NY can only win 0 or 1 games.
If NY wins 0, Bos wins ≥ 69 + 8 = 77

By winning all of their remaining games, Detroit can finish the season with a record of 76 and 86. If the Yankees win just 2 more games, then they will finish the season with a 77 and 85 record which would put them ahead of Detroit. So, let’s suppose the Tigers go undefeated for the rest of the season and the Yankees fail to win another game.

The problem with this scenario is that New York will have 8 games left with Boston, if the Red Sox win all of these games, they will end the season with at least 77 wins putting them ahead of the Tigers. Thus, the only way for Detroit to even have a chance of finishing in first place, is for New York to win exactly one of the 8 games with Boston and lose all their other games. Meanwhile, the Sox must lose all the games they play against teams other than New York. This puts them in a 3-way tie for first place. . .

Now let’s look at what happens to the Orioles and Blue Jays in our scenario. Baltimore has 2 games left with Boston and 3 with New York. So, if everything happens as described above, the Orioles will finish with at least 76 wins. So, Detroit can catch Baltimore only if the Orioles lose all their games to teams other than New York and Boston. In particular, this means that Baltimore must lose all 7 of its remaining games with Toronto. The Blue Jays also have 7 games left with the Yankees and we have already seen that for Detroit to finish in first place, Toronto must win all of these games. But if that happens, the Blue Jays will win at least 14 more games giving them at final record of 77 and 85 or better which means they will finish ahead of the Tigers. So, no matter what happens from this point in the season on, Detroit cannot finish in first place in the American League East.

How to use wax fans

Model:
Let $g_{i,j}$ be remaining games between $i$ and $j$
Let $w_i$ be # wins for $i$
Let $r_i = \sum_j g_{i,j}$ be # remaining games for $i$
Let $r_i = \sum_j g_{i,j}$ be the number of remaining games for $i$.

Can team $i$ come in first place? (tie is allowed)

- Team $i$ wins $\leq W_i + r_i$.

So, team $i$ can win $\leq (W_i + r_i) - W_i$.

Todo: assign winning team for each game.

Left = games, right = teams.

\text{\textbf{Diagram:}}

\text{\textbf{Proof:}}
Proof

"saturated \Rightarrow \text{ team } i \cup j" \Rightarrow \text{flow sat all edges.}

Let \( f \) be the flow sat all edges.

Assume \( f \) is integral.

Team \( i \) wins > \( g_{ij} \rightarrow t_i \), many games against team \( j \):

- all games are played (using sat) \( g_{ij} \leq t_i \).
- all team \( i \) wins \( \leq W_i + r_i - W_i \) games

"team i \Rightarrow sat"

For each game \( g_{ij} \),

\[
\begin{align*}
\text{send} & \quad s \rightarrow g_{ij} \rightarrow t_i \rightarrow t, \\
\text{if} & \quad t_i \text{ wins} \\
\text{else} & \quad s \rightarrow g_{ij} \rightarrow t_j \rightarrow t, \\
\text{else} & \quad \end{align*}
\]