

## Baseball Elimination

$49 + 27 = 76$

Team	W-L	Left	NY	BAL	BOS	TOR	DET
New York Yankees	75-59	28		3	8	7	3
Baltimore Orioles	71-63	28	3		2	7	4
Boston Red Sox	69-66	27	8	2		0	0
Toronto Blue Jays	63-72	27	7	7	0		0
Detroit Lions	49-86	27	3	4	0	0	

*Handwritten notes: Arrows point from 'w\_i' to the W-L column and from 'r\_i' to the Left column. A bracket groups the columns NY through DET, with 'g\_{i,j}' written above it.*

Question: Can Detroit come to first place?

Assume:

- no game can tie
- Every game happens.

Notice: Detroit wins  $\leq 76$  games.

$\Rightarrow$  NY can only win 0 or 1 games.

If NY wins 0, Bos wins  $\geq 69 + 8 = 77$

By winning all of their remaining games, Detroit can finish the season with a record of 76 and 86. If the Yankees win just 2 more games, then they will finish the season with a 77 and 85 record which would put them ahead of Detroit. So, let's suppose the Tigers go undefeated for the rest of the season and the Yankees fail to win another game.

The problem with this scenario is that New York still has 8 games left with Boston. If the Red Sox win all of these games, they will end the season with at least 77 wins putting them ahead of the Tigers. Thus, the only way for Detroit to even have a chance of finishing in first place, is for New York to win exactly one of the 8 games with Boston and lose all their other games. Meanwhile, the Sox must lose all the games they play against teams other than New York. This puts them in a 3-way tie for first place...

Now let's look at what happens to the Orioles and Blue Jays in our scenario. Baltimore has 2 games left with Boston and 3 with New York. So, if everything happens as described above, the Orioles will finish with at least 76 wins. So, Detroit can catch Baltimore only if the Orioles lose all their games to teams other than New York and Boston. In particular, this means that Baltimore must lose all 7 of its remaining games with Toronto. The Blue Jays also have 7 games left with the Yankees and we have already seen that for Detroit to finish in first place, Toronto must win all of these games. But if that happens, the Blue Jays will win at least 14 more games giving them a final record of 77 and 85 or better which means they will finish ahead of the Tigers. So, no matter what happens from this point in the season on, Detroit can not finish in first place in the American League East.

How to use max flow

Model:

Let  $g_{i,j}$  be remaining games between  $i$  and  $j$

Let  $w_i$  be # wins for  $i$

Let  $r_i = \sum_j g_{i,j}$  be # remaining games for  $i$

Let  $r_i = \sum_j g_{i,j}$  be # remaining games for  $i$

Can team 1 come in first place? (tie is allowed)

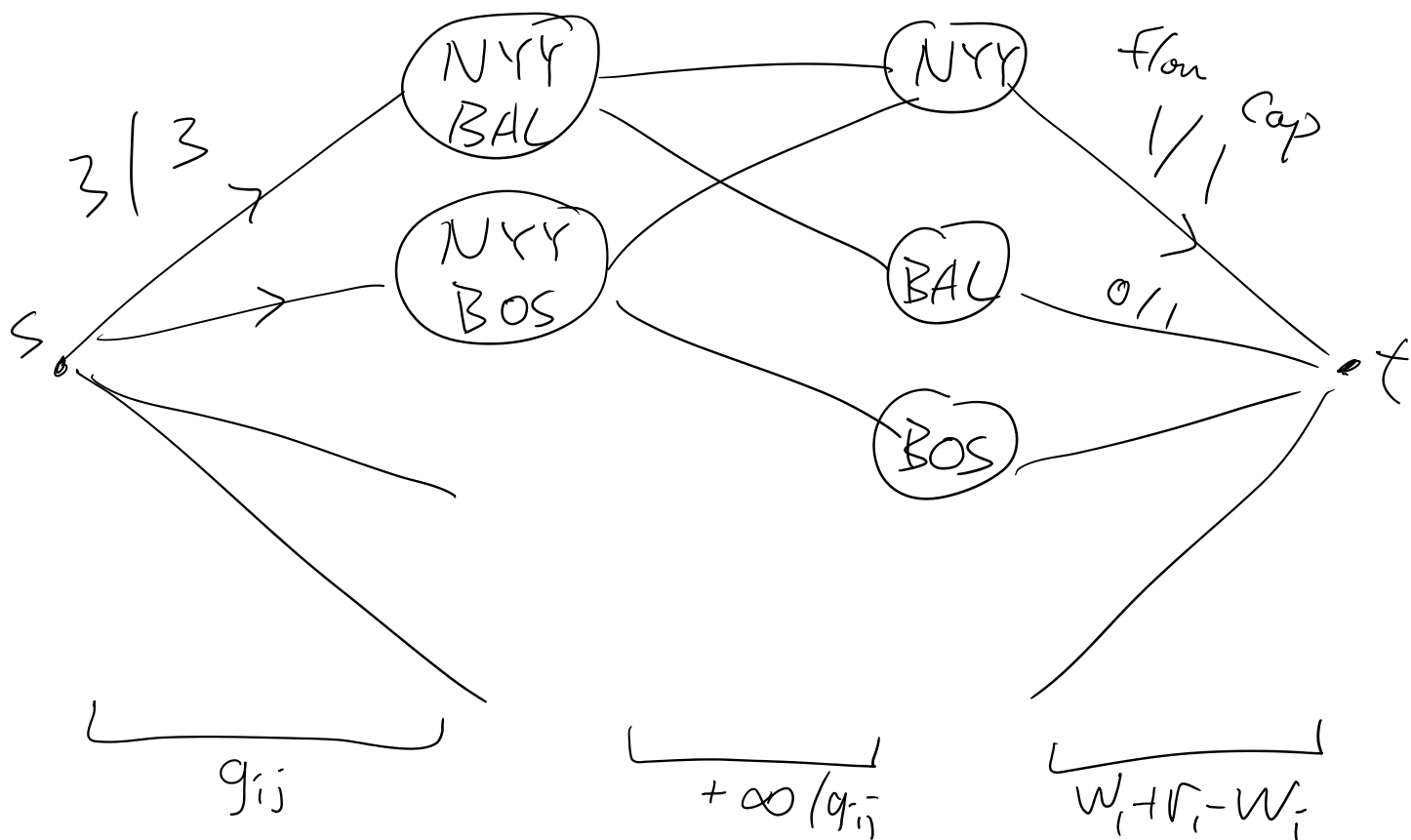
\* team 1 wins  $\leq W_1 + r_1$

So, team  $i$  can wins  $\leq (W_i + r_i) - W_i$  -1 if tie not allowed  
 many remaining games

Todo: assign winning team for each game.

Left = games

right = teams



Thm team 1 can be 1st place if and only if there is a flow saturated all edges out of  $s$

Proof

...  $\rightarrow \rightarrow$

## Proof

"saturated  $\Rightarrow$  team 1  $\checkmark$ "

Let  $f$  be the flow sat all edges.

Assume  $f$  is integral.

Team  $i$  wins  $f_{g_{ij} \rightarrow t_i}$  many games against team  $j$

- all games are played (using sat)
- all team  $i$  wins  $\leq W_i + r_i - W_i$  <sup>remaining</sup> games

"team 1  $\Rightarrow$  sat"

For each game  $g_{ij}$ ,

send  $s \rightarrow g_{ij} \rightarrow t_i \rightarrow t$

if  $t_i$  wins

else

$s \rightarrow g_{ij} \rightarrow t_j \rightarrow t$

else

