Lemma

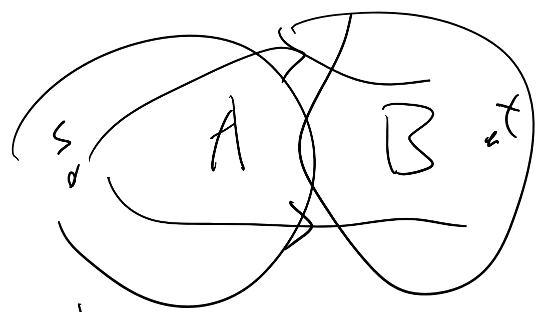
Let f be any s-t flow, let $(\underset{s}{A}, \underset{t}{B})$ be s-t cut

Then

$$\underbrace{\sum_{e \text{ out of } A} f(e) - \sum_{e \text{ into } A} f(e)}_{\text{net flow across } (A,B)} = \underbrace{v(f)}_{\text{flow value}}$$

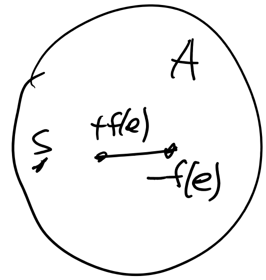
Proof

$$v(f) \overset{\boxed{\text{def}}}{=} \sum_{e \text{ out of } s} f(e) - \overbrace{\sum_{e \text{ into } s} f(e)}^{0}$$



flow conservation

$$= \left(\sum_{e \text{ out of } s} f(e) - \sum_{e \text{ into } s} f(e)\right) + \sum_{\substack{v \neq s \\ v \in A}} \overbrace{\left(\sum_{e \text{ out of } v} f(e) - \sum_{e \text{ into } v} f(e)\right)}^{0}$$



$$= \sum_{e \text{ out of } A} f(e) - \sum_{e \text{ into } A} f(e)$$

Def (A,B) is s-t cut

if A, B partition V $\qquad (A, A^c)$

and $s \in A$, $t \in B$.

Weak duality

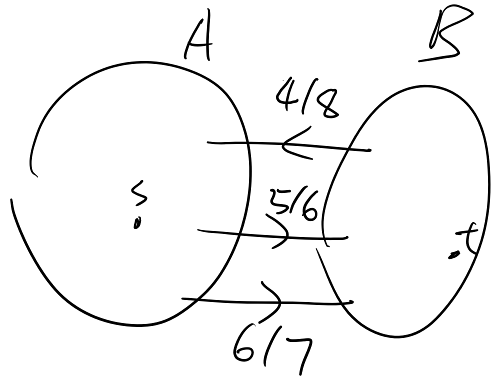
For any s-t flow f and s-t cut (A,B)

$$v(f) \leq \text{cap}(A,B)$$

for any s-t

$$v(f) \le cap(A,B)$$

Proof



$$v(f) = \sum_{e \text{ out of } A} f(e) - \sum_{e \text{ into } A} f(e)$$

$$\le \sum_{e \text{ out of } A} f(e)$$

$$Cap(A,B) = 6+7$$

$$\le \sum_{e \text{ out of } A} c(e) = Cap(A,B) \qquad v(f) = 5+6-4 = 7$$

Corollary If there is s-t cut (A,B) and s-t flow f

st $v(f) = cap(A,B)$

Then (A,B) is min s-t cut

f is max s-t flow

Pf If there A', B' st $cap(A',B') < cap(A,B)$

Then $v(f) \le cap(A',B') < cap(A,B) = v(f)$

it is contradiction.

$$T(n) = T(n-1) + T\left(\frac{n}{2}\right) + O(n^{2018})$$

$$= T(n-2) + T\left(\frac{n}{2}\right) + T\left(\frac{n-1}{2}\right) + O(n^{2018})$$

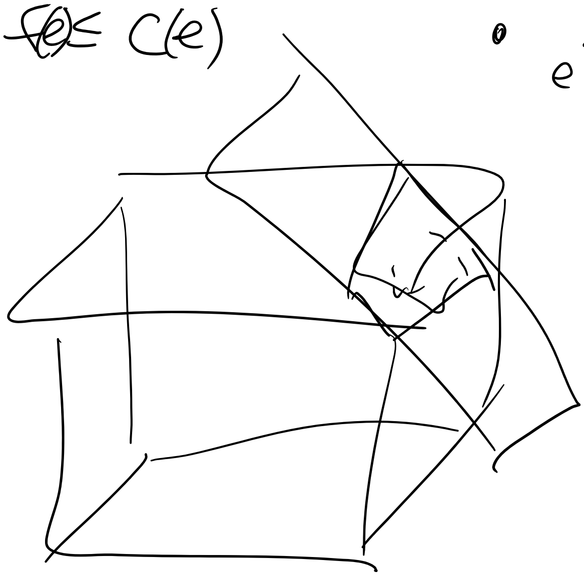
$$\vdots$$

$$= \underbrace{T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + \cdots T\left(\frac{n}{4}\right)} + O(n^{2019})$$

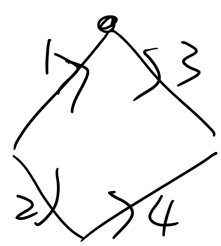
$$\begin{cases} \geq \frac{n}{2} T\left(\frac{n}{2}\right) + O(n^{2019}) \\ \leq \frac{n}{2} T\left(\frac{n}{4}\right) + O(n^{2019}) \end{cases}$$

How to draw s-t flow

- $0 \leq \cancel{f(e)} \leq c(e)$
- $\sum_{e \text{ out } v} f(e) = \sum_{e \text{ in } v} f(e)$



$\forall v \notin \{s, t\}$



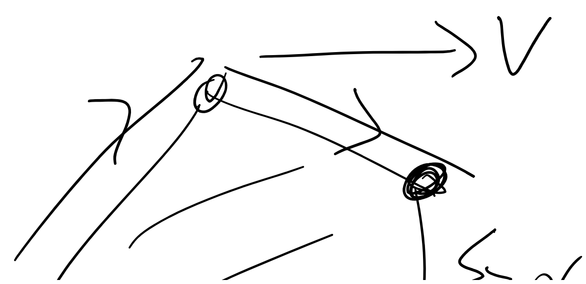
$f_1 = f_3$

$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}^T (f) = 0.$

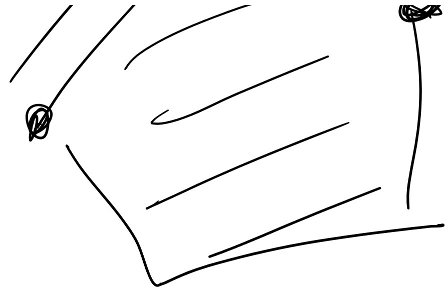
$f = \begin{pmatrix} \\ \\ \\ \end{pmatrix} \rightarrow |E|$ many coordinate

What is max flow?

$\max v^T f$



$f \in$ 



Simplex

linear program