

Then The matching M^* G-S algo outputs is man-optimal.

Proof by contradiction:

(★ reject includes the case of trading up)

Not man-optimal \Rightarrow Some men rejected by his valid partner.

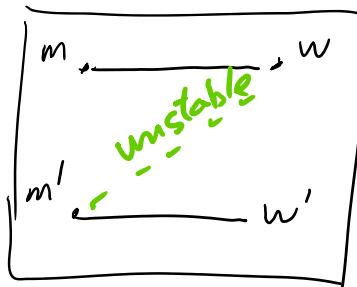
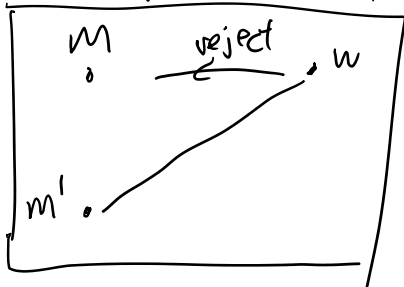
Let m be the first man with such rejection.

w be the one who reject.

M be a stable matching st $(m, w) \in M$

(At the time of rejection) M^*

M



Let m' is the current partner of w at rejection time

w' is the partner of m' in M

We know

- w likes $m' > m$ (due to rejection)
- m' likes $w > w'$

(If m' like w' more,
 m' propose to w' first.
 No rejection because (m, w) is first rejection)

$\Rightarrow (m', w)$ is unstable

\Rightarrow Contradicts the fact M is stable

TL Th. M^* is man-optimal

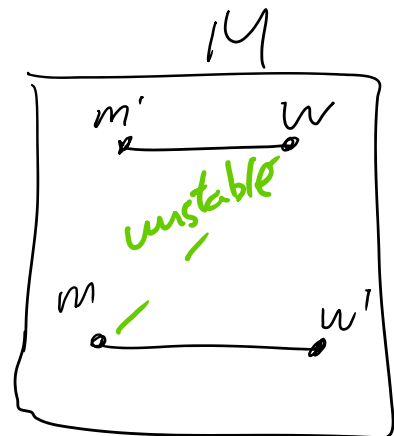
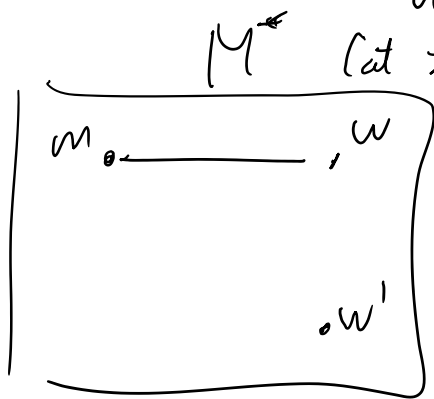
Thm The matching M^* G-S algo outputs is woman-preferred

Proof by contradiction

Not woman-preferred \Rightarrow

$(m, w) \in M^*$ and m is not the worst
valid partner for w

\exists stable M st
 $(m', w) \in M$
 w likes $m > m'$



We know

- w likes $m > m'$ (due to def)
- m likes $w > w'$ (M^* is man-optimal)

Hence (m, w) is unstable in M .