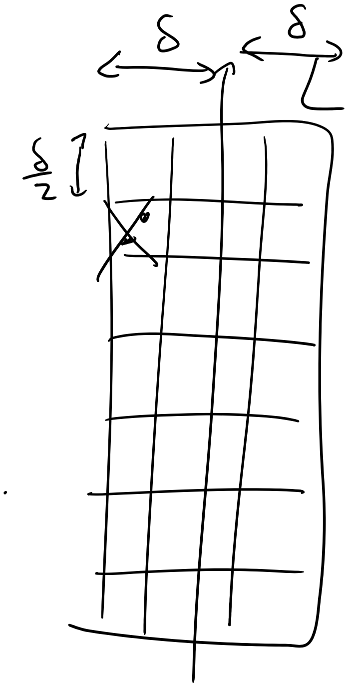


Claim: No 2 points lie in the same box

Pf: If 2 points in the same box,

then distance $\le \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2} < \delta$

(contradicts to the def of δ).

$ALG(p_1, \dots, p_n)$

find vertical line cut $\{p_1, \dots, p_{\frac{n}{2}}\}$ on left

$\{p_{\frac{n}{2}+1}, \dots, p_n\}$ on right

$\delta_1 = ALG(p_1, \dots, p_{\frac{n}{2}})$

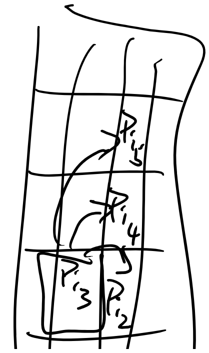
$\delta_2 = ALG(p_{\frac{n}{2}+1}, \dots, p_n)$

$\delta = \min(\delta_1, \delta_2)$ ⟵ we know there is a pair with dist exactly δ

Let L be strip.

Let $p_{i_1}, p_{i_2}, \dots, p_{i_m}$ be the points in L.

Sort the p_{i_j} according the y axis



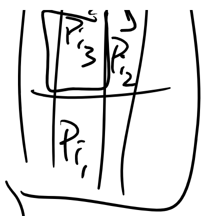
for $j = 1, \dots, m$

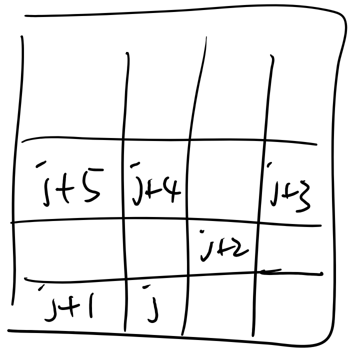
for $k = 1, 2, \dots 11$

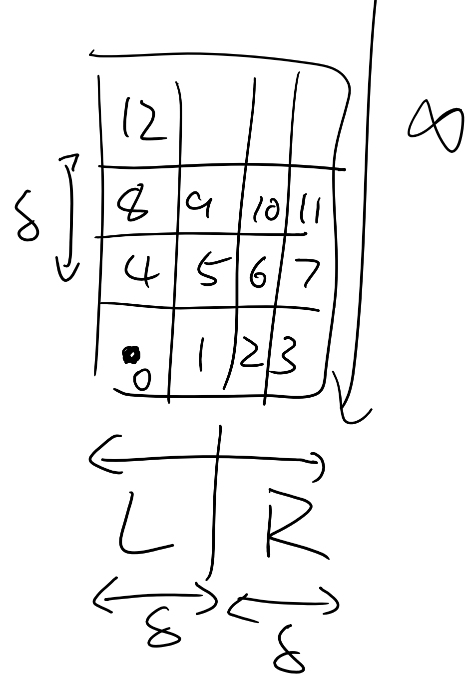
for $k = 1, 2, \ldots, 11$

    if $j + k \le m$

        $\delta = \min(\delta, dist(P_{i_j}, P_{i_{j+k}}))$







Runtime:

$$T(n) \le \begin{cases} 1 & \text{if } n = 1 \\ 2T(\frac{n}{2}) + O(n\log n) + O(n) & \text{else.} \end{cases}$$

$$T(n) = O(n\log n) + 2T(\tfrac{n}{2})$$
$$= O(n\log n) + 2\left[O(\tfrac{n}{2}\log\tfrac{n}{2}) + 2T(\tfrac{n}{4})\right]$$
$$= O(n\log n) + O(n\log n) + 4T(\tfrac{n}{4})$$

$\log n$ levels

$\vdots$

$= O(n \log^2 n)$

Correctness:

Let $p_i, p_{i'}$ be the closest pair in the strip L with dist $\leq \delta$.

WLOG, $i < i'$

Claim $i' \leq i + 11$

$p_{i'}$ must appear in the same/next/next next row as p_i

- There are 12 boxes in those 3 rows.
- Each box contain 1 point

$\Rightarrow i' \leq i + 11$