Lemma: binary tree $T \rightarrow$ Huffman Tree $H$ via inversion.

Proof by induction.

I: At $k^\text{th}$ iteration of Huffman, all nodes in the Q is a subtree of $T$ (after inversions).

Base case: all nodes in Q are leaves of $T$.

IS: Huffman picks $A, B$ from $Q$ and form a new tree in $Q$.

By I, we know, $A$ and $B$ are sub tree of $T$ (after...)

Case 1: $A, B$ are siblings in $T$.

Then $N$ is a subtree of $T$.

No swapping is needed.

Case 2: $A, B$ are at sib in $T$.

WLOG, depth($A$) $\geq$ depth($B$)

Let $C$ be sib of $A$.

Want to swap $C, B$.

Note that

- $\text{Freq}(C) \geq \text{Freq}(B)$ (Huffman picks $2\text{min}$)
- $\text{depth}(C) > \text{depth}(A) > \text{depth}(B)$
- \text{size}(C) = \text{size}(A)$ (minimum size within $B$)
- $\text{depth}(C) = \text{depth}(A) > \text{depth}(B)$

So, $C, B$ are inverses.

After swap, $N$ is a subtree of $T$.