Lecture 11

Friday, October 19, 2018 1:17 PM

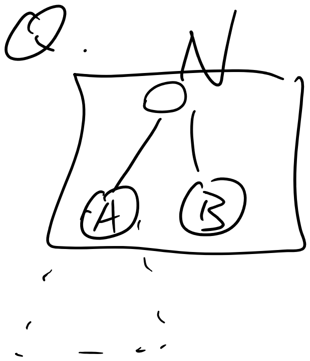
Lemma binary tree $T \longrightarrow$ Huffman Tree H via Traversing

Proof by induction.

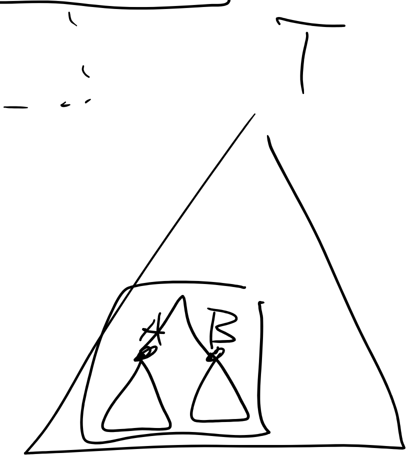
IH: At k^{th} iteration of Huffman,
all nodes in the Q is a subtree of T (after traversing)

Base case: all nodes in Q are leaves of T.

IS: Huffman picks A, B from Q.
and form a new tree in Q



By IH, we know, A and B are
subtree of T. (after ...)



Case 1: A, B are siblings in T.

Then N is a subtree of T.

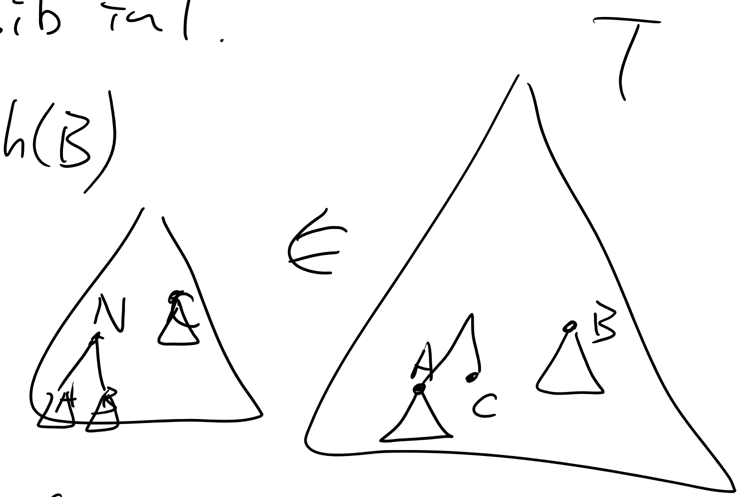
No swapping is needed ✓

Case 2: A, B are not sib in T.

WLOG, $\mathrm{depth}(A) \geq \mathrm{depth}(B)$

Let C be sib of A

Want to swap C, B.



Note that

- $\mathrm{freq}(C) \geq \mathrm{freq}(B)$ (Huffman picks 2 min)
- $\mathrm{depth}(C) = \mathrm{depth}(A) \geq \mathrm{depth}(B)$

- freq(C) ≥ freq(B) (Huffman picks 2 min)
- depth(C) = depth(A) ≥ depth(B)

So, C, B are inversion.

after swap, N is a subtree of T ✓