Lecture 10

Wednesday, October 17, 2018 1:15 PM

Cut property

Let e be the min cost edge in a cut $(S, V-S)$.
Let T^* be any MST. Then $e \in T^*$.

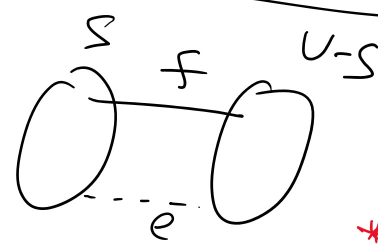
Wrong Proof: (by contradiction)

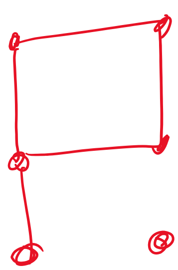
Suppose $e \notin T^*$.

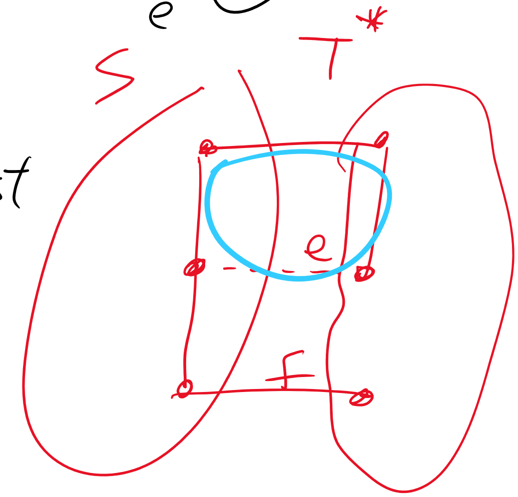
$\exists f \in T^*$ st $f \in (S, V-S)$.

By def e, $C_e < C_f$.

So, $T' = T^* - \{f\} + \{e\}$ has a lower cost







Proof: (by contradicts)

Suppose $e \notin T^*$. Add e into T^*.

$T^* + e$ is not tree.

Get a cycle C. which cross $(S, V-S)$

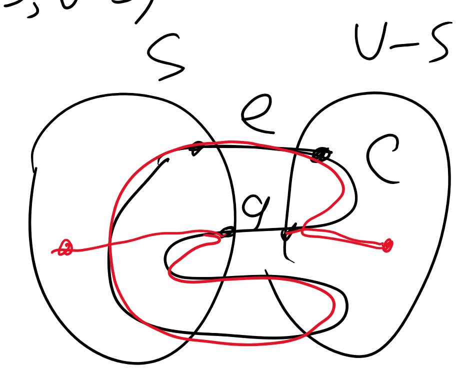
Let $g \in C$ st $g \neq e$.

$g \in (S, V-S)$.

Let $T = T^* - \{g\} + \{e\}$.

Since $C_e < C_g$, so, T has smaller cost.

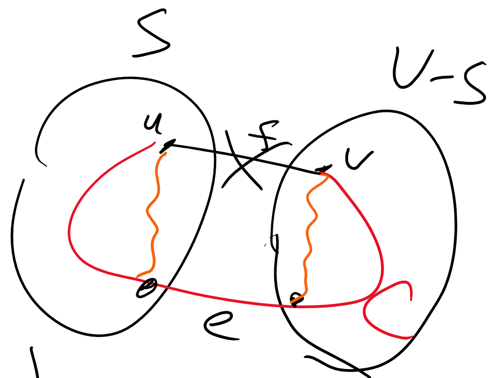
T is spanning tree



T is spanning tree
- T covers n vertices. ✓
- T has n-1 edge ✓
- T is connected since both end pts of g ✓ is covered by rest of C.

Cycle property Let f be max cost edge in cycle C. Then $f \notin T^*$ for any MST T^*.

Proof by contradiction.



Suppose $f \in T^*$.

Consider removing f from T^*

It creates 2 connected components

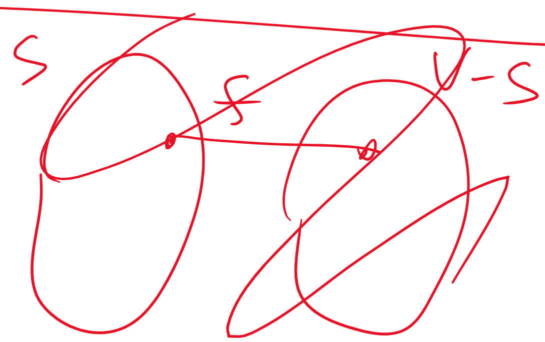
(S is the connected components containing u).

Let $T = T^* - \{f\} + \{e\}$.

Again T is spanning tree.

Since $c_e < c_f$, so cost $T <$ cost T^*

(contradict).



T is s tree

T is a tree 

$\Rightarrow$ $|T| = n-1$ ✓

T is connected ✓

T covers every vertices.