CSE 421

Edmonds-Karp Algorithm, More Applications

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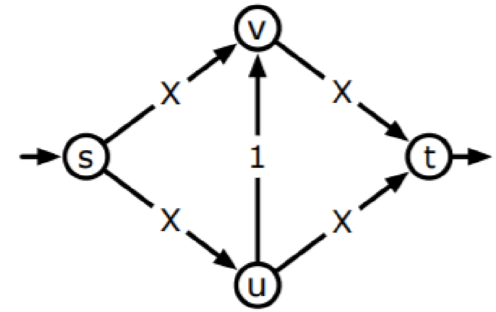
FF may converge to wrong answer



- FF is not polynomial time.

It can take X steps for this graph.

- FF may not even converge to a correct flow for irrational capacity.

$\phi = \frac{\sqrt{5}-1}{2}$, $X = 3$.

Augmenting paths along

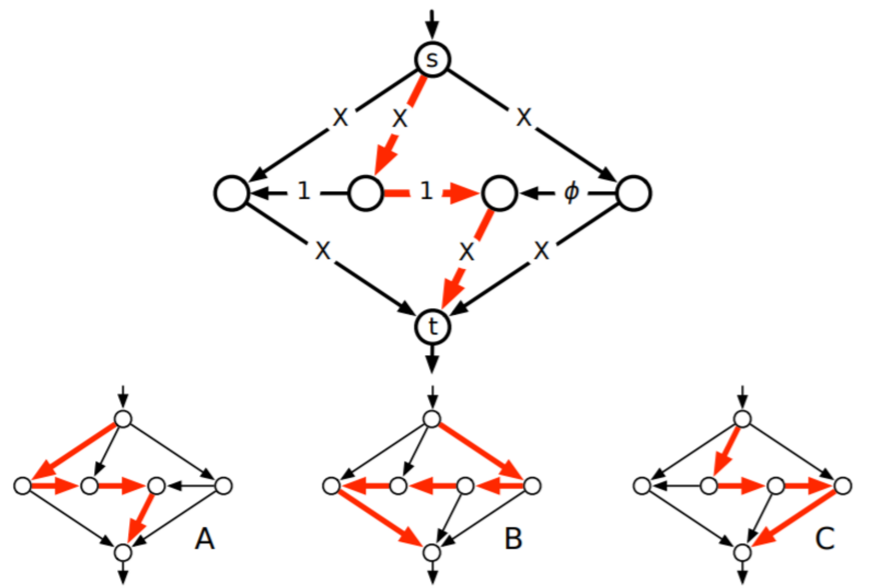
the large figure.

Then,

B,C,B,A,B,C,B,A,B,C,B,A,…

In limit, it will send $4 + \sqrt{5}$ unit.

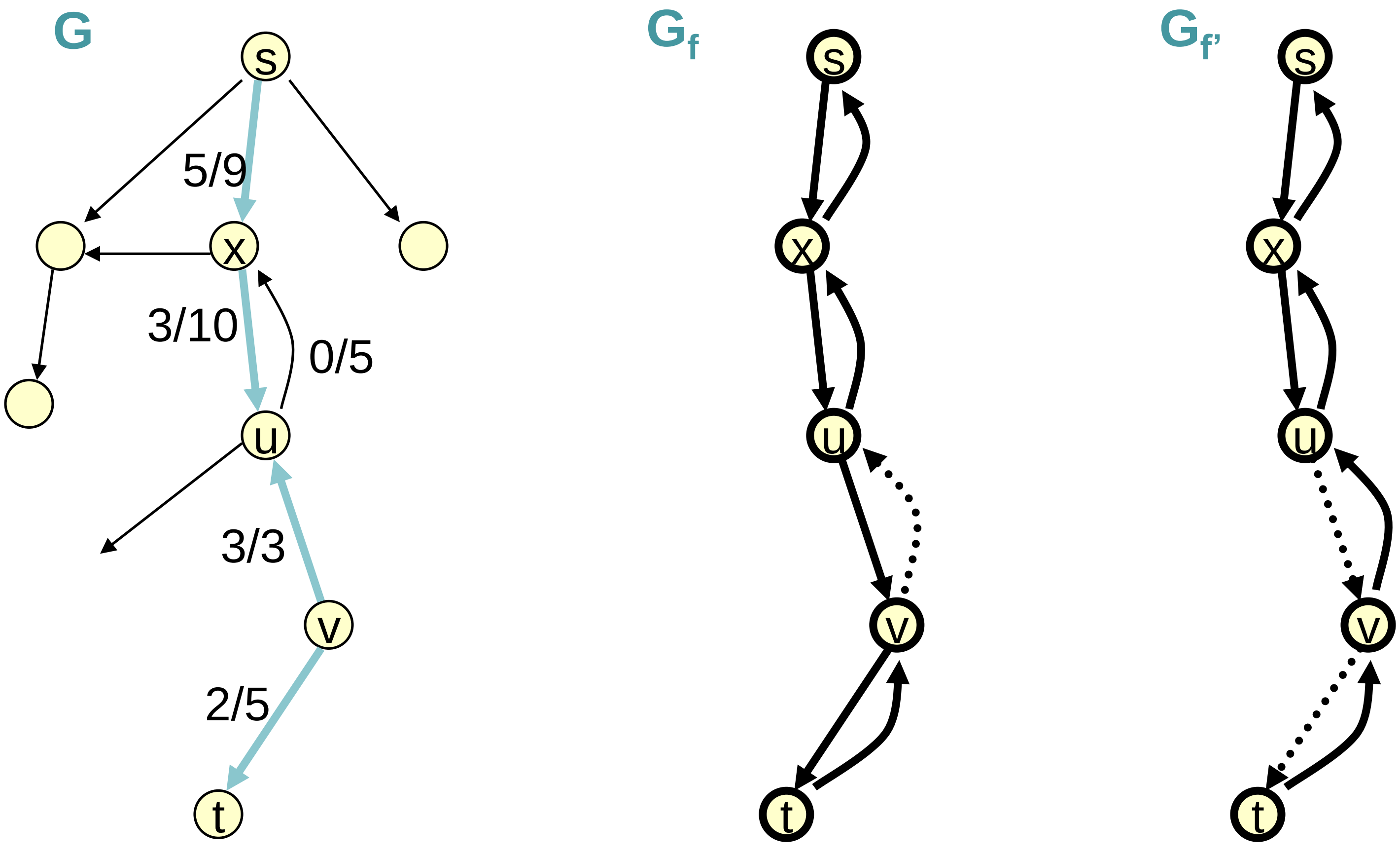
But, maxflow = 7.



Uri Zwick's non-terminating flow example, and three augmenting paths.

Edmonds-Karp Algorithm

- Use a shortest augmenting path
  (via Breadth First Search in residual graph)
- Time: $O(m^2 n)$.



Distance to s is non-decreasing.

Let f be a flow, G_f the residual graph, and P a shortest augmenting path. Then no vertex is closer to s after augmentation along P.

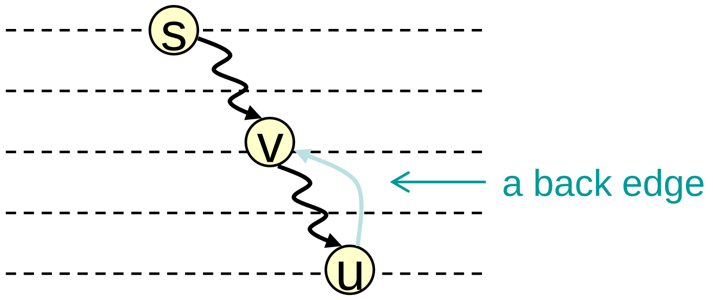
Proof: Augmentation along P only

- deletes forward edges

  no new (hence no shorter) path created

- adds back edges that go to previous vertices along P

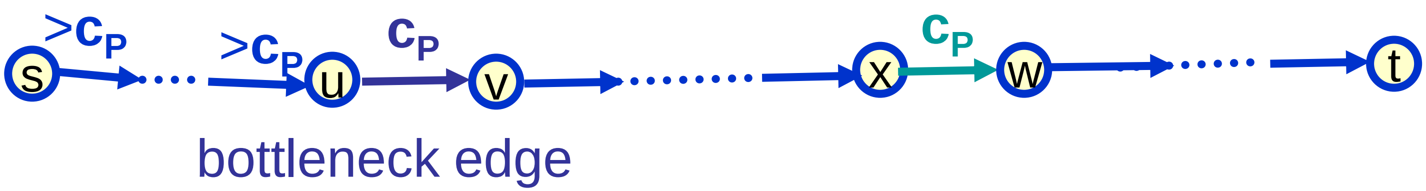
  BFS is unchanged, since v visited before (u, v) examined



Distance for bottleneck edges

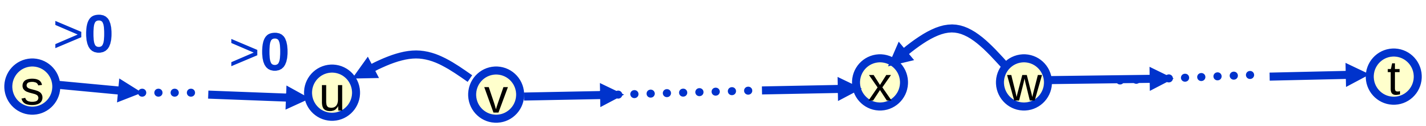
Let $d_f(s,v)$ be the distance from s to v on G_f.

Shortest s-t path **P** in **G$_f$**

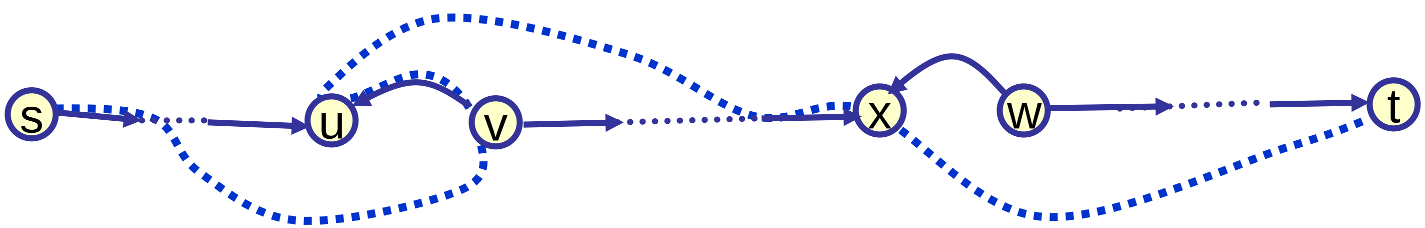


After augmenting along P

$d_f(s,v) = d_f(s,u) + 1$ since this is a shortest path



For (u,v) to be bottleneck again for some flow f'



$$d_{f'}(s,u) = d_{f'}(s,v) + 1 \geq d_f(s,v) + 1 = d_f(s,u) + 2$$

Theorem

Edmonds-Karp performs $O(mn)$ flow augmentations

Proof:

- Each step, some edge disappear from G_f.
  (Note however that some edge may reappear.)
- Any edge (u, v) disappears from G_f at most $n/2$ times.
  (because the distance increased by 2 every disappearance.)
- There are at most $mn/2$ disappearances.

Total time is $O(m^2n)$.

Maximum flow

Current Best (U = maximum capacity):

- $O((m + nF)\log^{O(1)}(nU))$ [Karger-Levine 02]
- $O(mn)$ [Orlin 13]
- $O(m\sqrt{n}\log^{O(1)}(nU))$ [Lee-Sidford 13]
- $O(m^{\frac{10}{7}}U^{\frac{1}{7}}\log^{O(1)}(nU))$ [Madry 13]
- $O((m + m^{\frac{3}{4}}n^{\frac{1}{4}}\sqrt{F})\log^{O(1)}(nU))$ [Sidford-Tian 17]