CSE421: Introduction to Algorithms

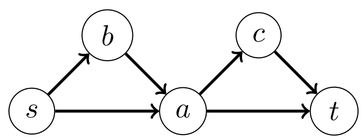
Homework 9

Due: December 5, 2018

1. Prove that the HW 7 Problem 1 is NP-complete, i.e. the decision problem of determining whether there is a subset with total calories K fulfilling the target values is NP-complete. (This justifies the dependency on α, β, γ in the time complexity in the HW7 Problem 1.)

   NOTE: The Subset Sum problem is NP-complete. In the Subset Sum problem, we are given positive numbers $w_1, \ldots, w_n$, and we want to know if there is a subset that adds up to exactly W.

2. Given a directed graph $G = (V, E)$, a pair of vertices s, t and an integer k. We want to output yes if there are k *vertex* disjoint paths from s to t and no otherwise. For example, in the following graph there are two edge disjoint paths from s to t but no two vertex disjoint paths from s to t. Design a polynomial time

   algorithm for this problem.

3. Give a polynomial time algorithm to find the minimum vertex cover in a bipartite graph.

   HINTS:

   (a) Construct a flow network from the input bipartite graph just as in the maximum matching algorithm.

   (b) Show that every min-cut in this flow network gives a vertex cover whose size is the same as the capacity of the cut.

   (c) Show that every minimum sized vertex cover in the bipartite graph gives a cut whose capacity is the same as the size of the vertex cover.

   (d) Write down the algorithm and prove that it works.

4. **Extra Credit:** Consider a random bipartite graph $G = (X \cup Y, E)$ with $|X| = |Y| = n$ constructed as follows: For each vertex $x \in X$, we connect x to random $2018 \log(n)$ many distinct vertices in $y \in Y$.

   Prove that G has a perfect matching with probability $\Omega(1)$.