CSE421: Introduction to Algorithms

Homework 2

Due: October 10, 2018

Unless specified otherwise, all graphs are undirected.

1. Arrange the following in increasing order of asymptotic growth rate. No justification is needed.
   $f_1(n) = 4n^4$, $f_2(n) = \frac{n^5}{2}$, $f_3(n) = n(\log(n))^{100}$, $f_4(n) = 2^{n \log n}$,
   $f_5(n) = 2^{\sqrt{n}}$, $f_6(n) = 2^{\log(n)^{0.9}}$, $f_7(n) = n^{1/\sqrt{\log n}}$, $f_8(n) = 2^{0.9 \log n}$.

2. Prove that in any tree with n vertices, the number of nodes with degree 8 or more is at most $(n-1)/4$.

3. Prove that in every graph with at least 2 vertices, there must be two vertices that have the same degree.

4. Given a graph $G = (V, E)$ with n vertices and m edges. Given two vertices s and t in G. Assume all edges are unit length. Give an algorithm that computes the number of shortest paths between s and t in G in time $O(n + m)$. Justify the running time bound of your algorithm.

   (Note that there can be exponentially many shortest paths between s and t. Therefore, it is impossible to list out all these paths in linear time.)

5. **Extra Credit:** Given a tree T with n vertices. Give an $O(n)$ time algorithm that finds a vertex v such that all connected components of $T - v$ has at most $n/2$ vertices. Justify the running time bound of your algorithm.