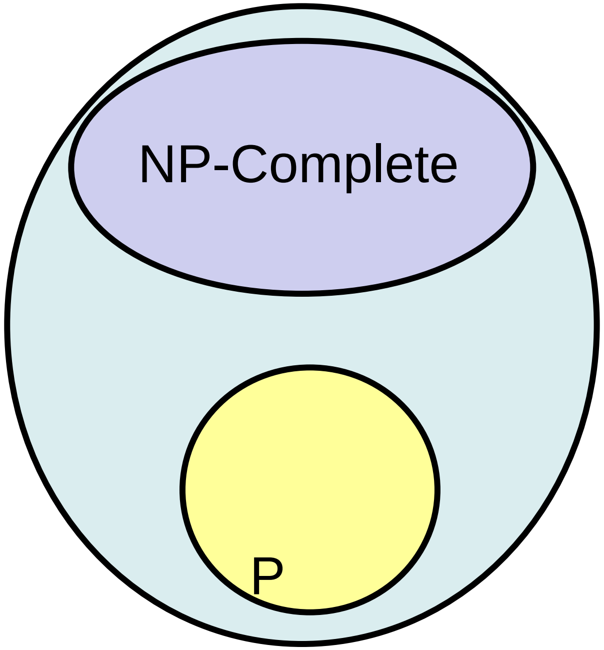


CSE 421 Algorithms

Richard Anderson

Lecture 27

Survey of NP Complete Problems

Announcements

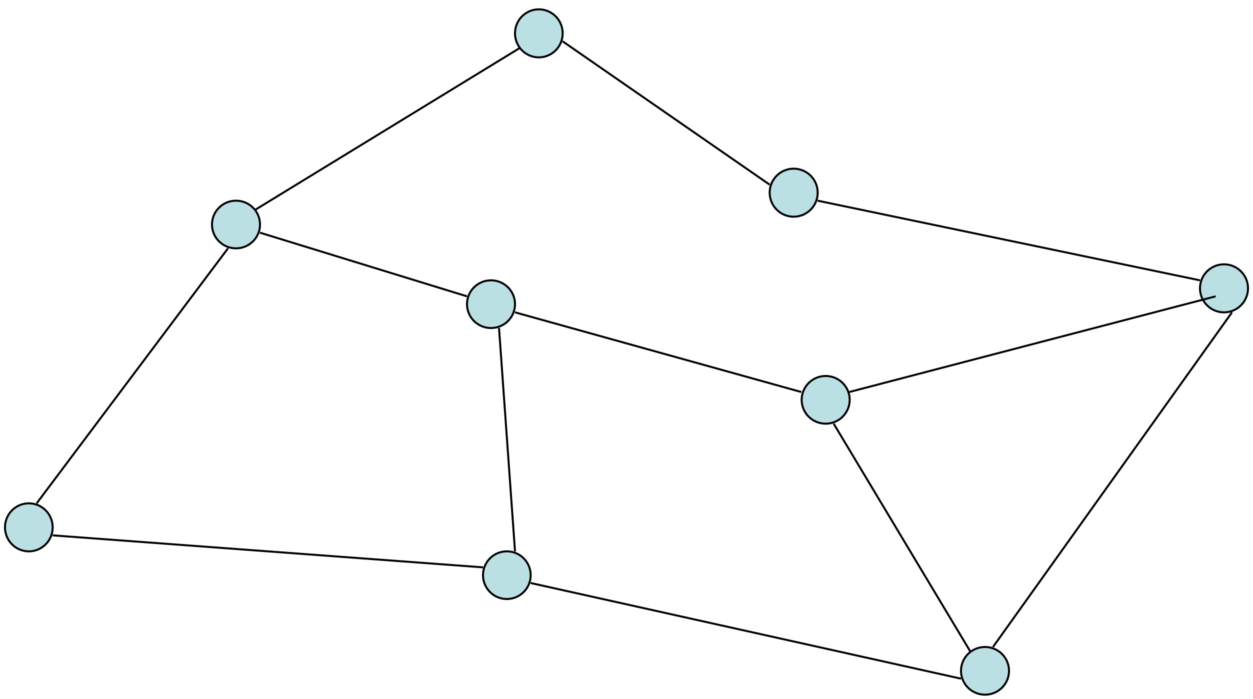
- Final exam,
  - Monday, December 12, 2:30-4:20 pm
  - Comprehensive (2/3 post midterm, 1/3 pre midterm)
- Review session
  - TBD
- Online course evaluations available

NP Complete Problems

1. Circuit Satisfiability
2. Formula Satisfiability
   a. 3-SAT
3. Graph Problems
   a. Independent Set
   b. Vertex Cover
   c. Clique
4. Path Problems
   a. Hamiltonian cycle
   b. Hamiltonian path
   c. Traveling Salesman
5. Partition Problems
   a. Three dimensional matching
   b. Exact cover
6. Graph Coloring
7. Number problems
   a. Subset sum
8. Integer linear programming
9. Scheduling with release times and deadlines

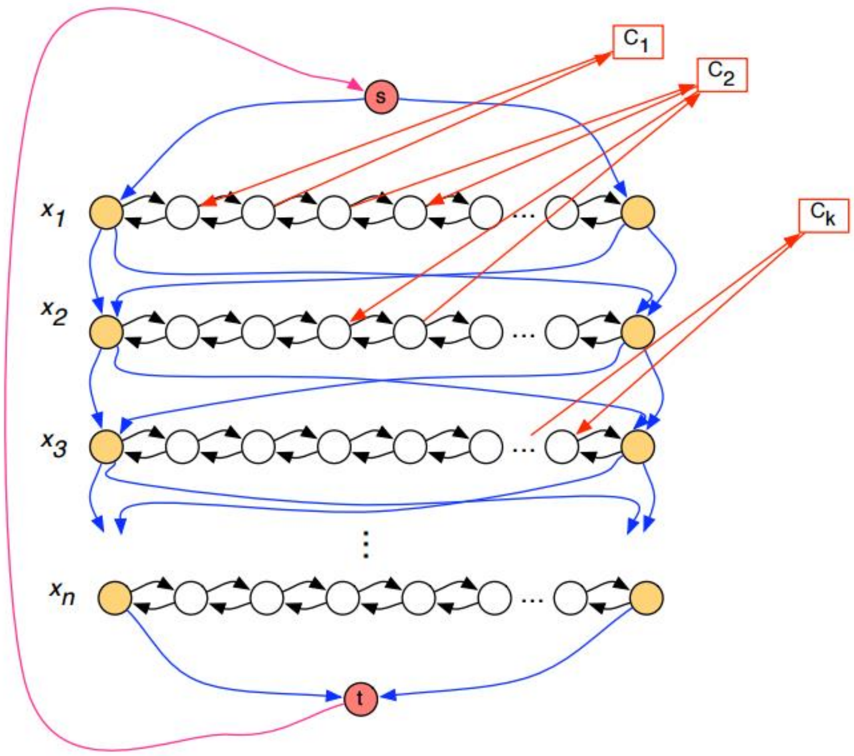
Hamiltonian Circuit Problem

- Hamiltonian Circuit – a simple cycle including all the vertices of the graph



Thm: Hamiltonian Circuit is NP Complete

- Reduction from 3-SAT



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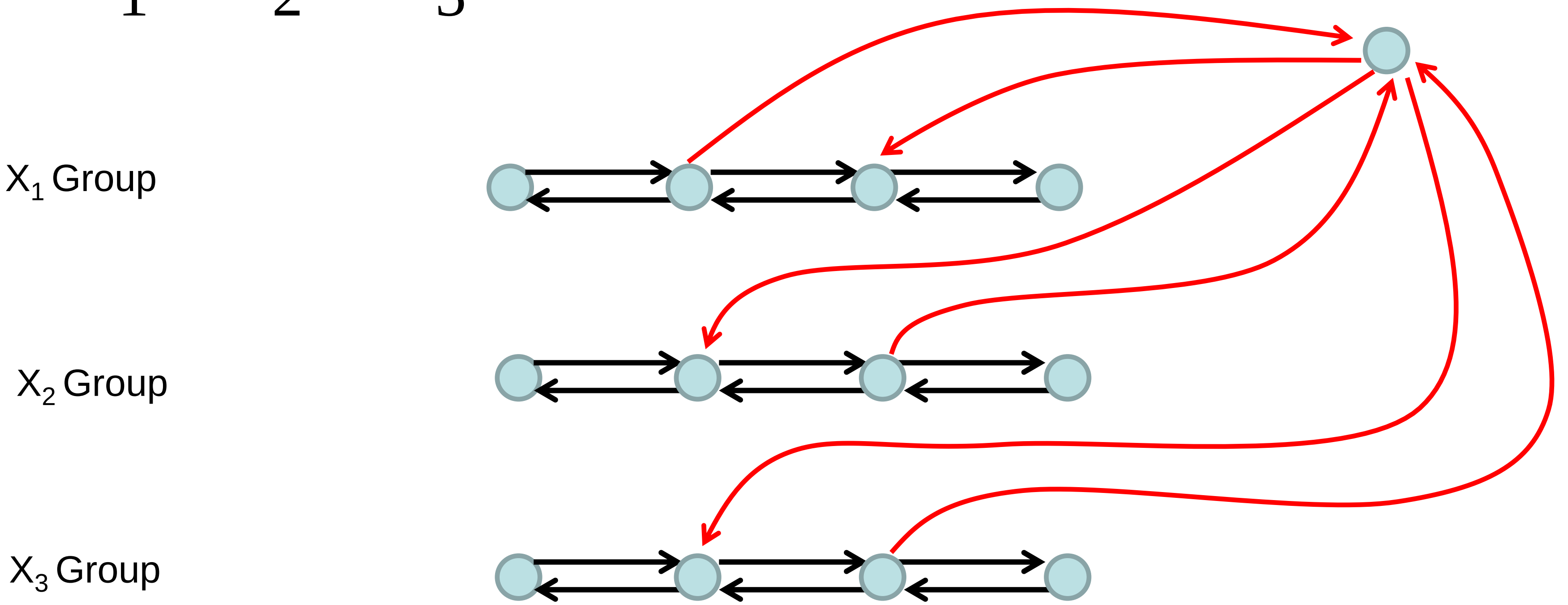
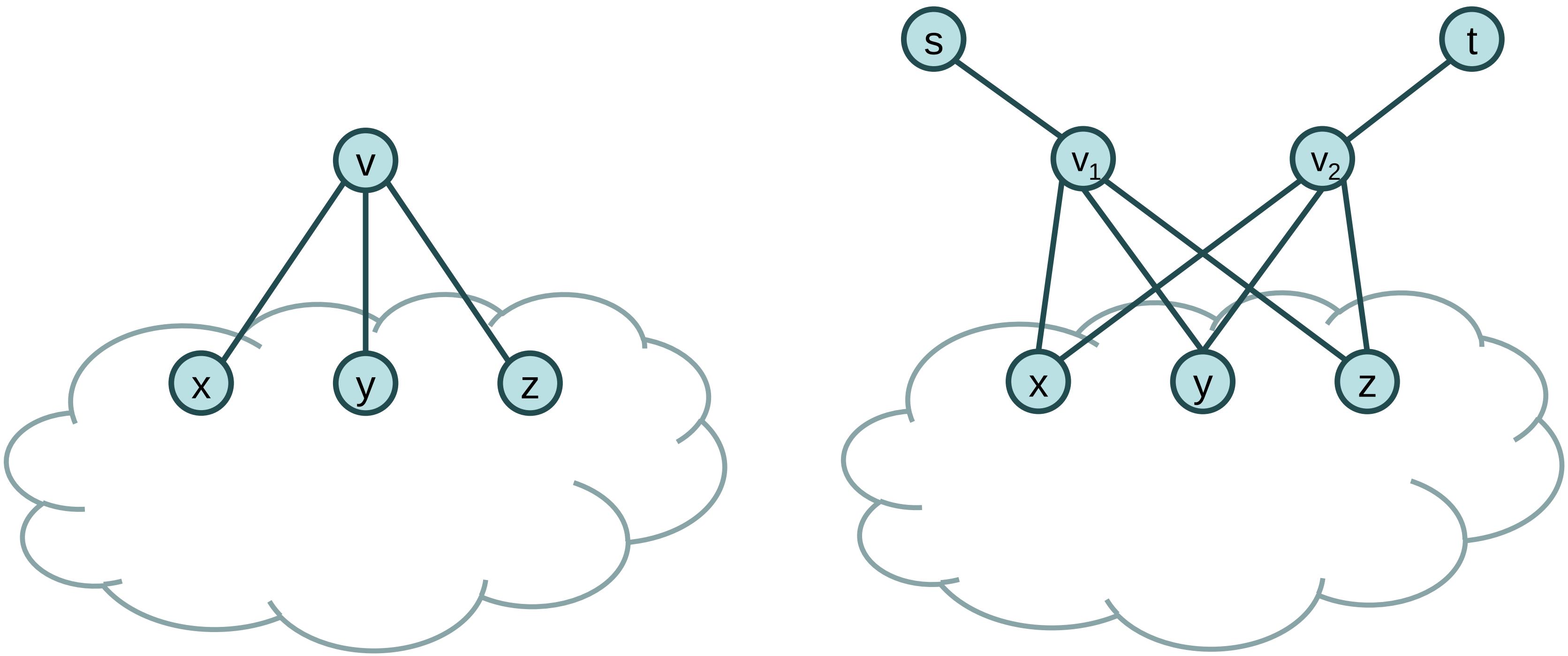
Clause Gadget

$$x_1 \vee \overline{x_2} \vee \overline{x_3}$$



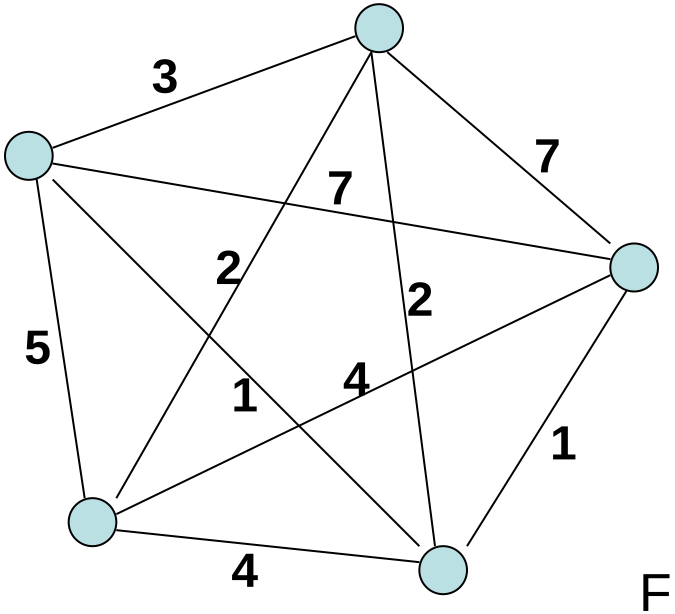
Reduce Hamiltonian Circuit to Hamiltonian Path

G_2 has a Hamiltonian Path iff G_1 has a Hamiltonian Circuit



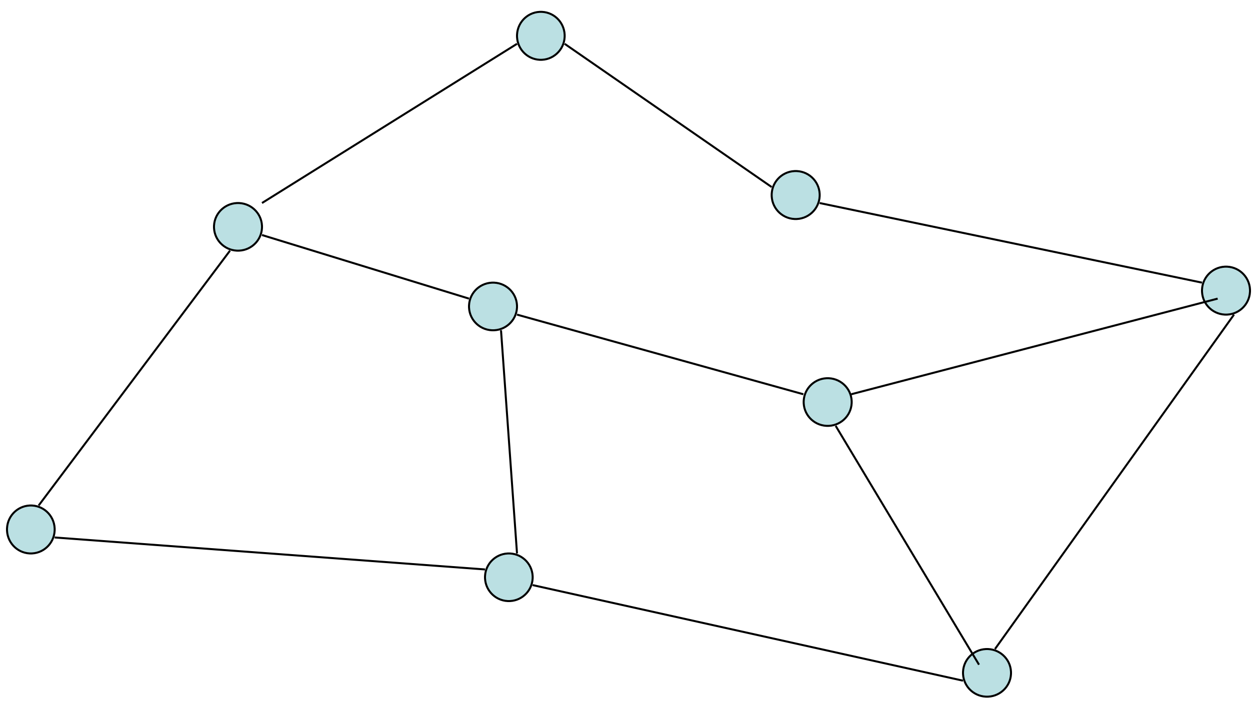
Traveling Salesman Problem

- Given a complete graph with edge weights, determine the shortest tour that includes all of the vertices (visit each vertex exactly once, and get back to the starting point)

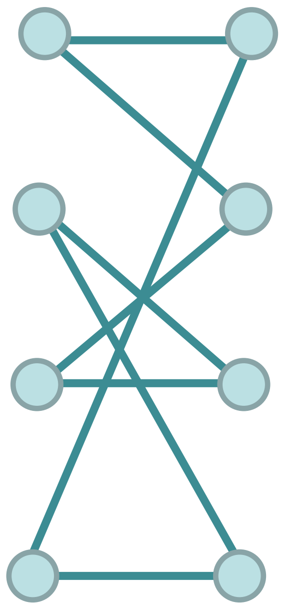


Find the minimum cost tour

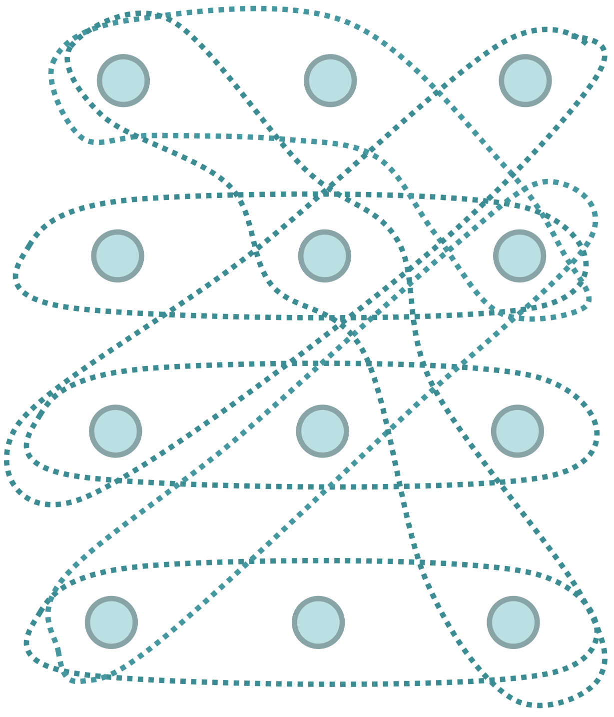
Thm: HC $<_P$ TSP



Matching

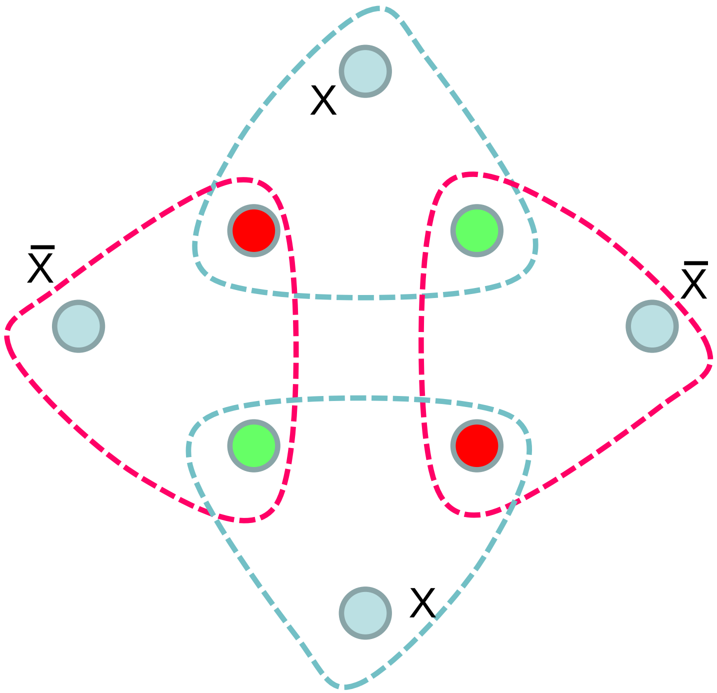


Two dimensional matching

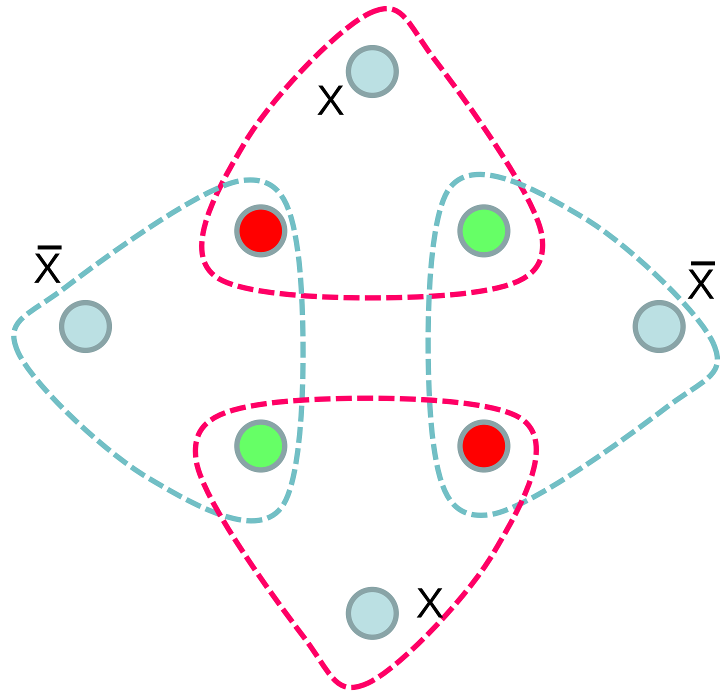


Three dimensional matching (3DM)

3-SAT $<_P$ 3DM



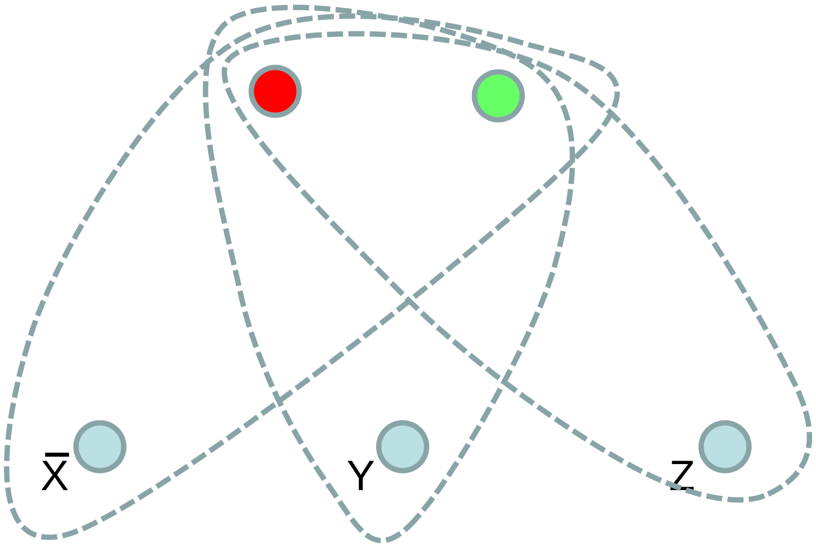
X True



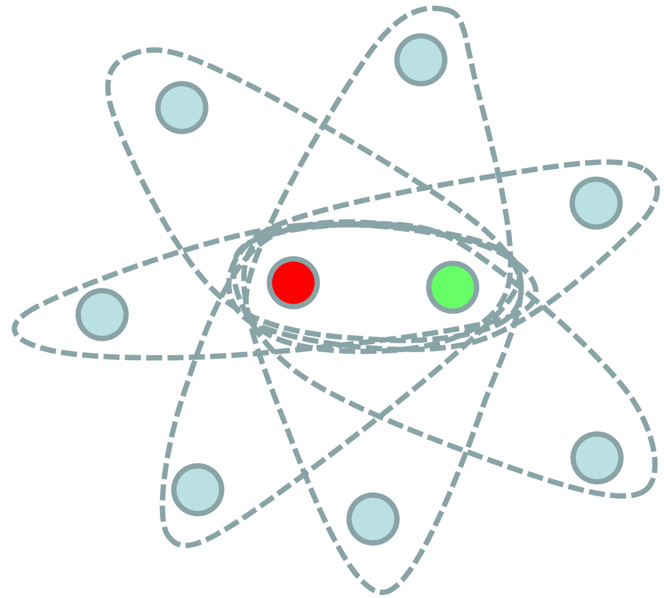
X False

Truth Setting Gadget

3-SAT $<_P$ 3DM



Clause gadget for (X̄ OR Y OR Z)



Garbage Collection Gadget
(Many copies)

Exact Cover (sets of size 3) XC3

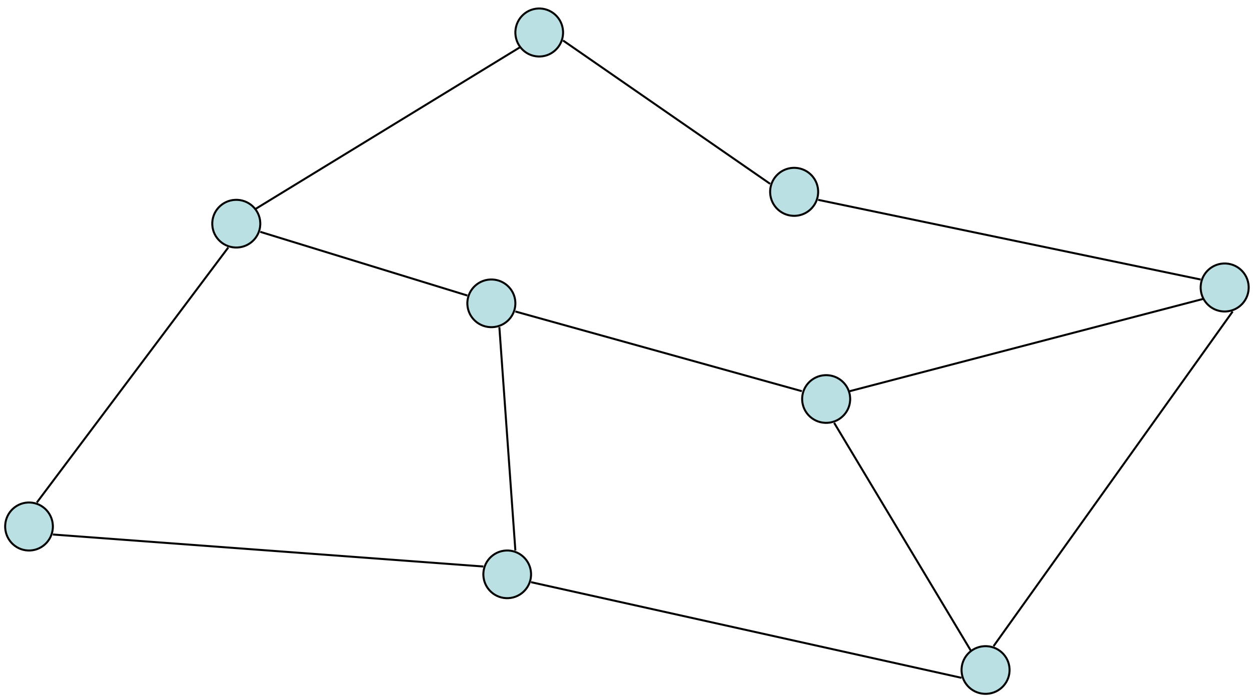
Given a collection of sets of size 3 of a domain of size 3N, is there a sub-collection of N sets that cover the sets

(A, B, C), (D, E, F), (A, B, G),
(A, C, I), (B, E, G), (A, G, I),
(B, D, F), (C, E, I), (C, D, H),
(D, G, I), (D, F, H), (E, H, I),
(F, G, H), (F, H, I)

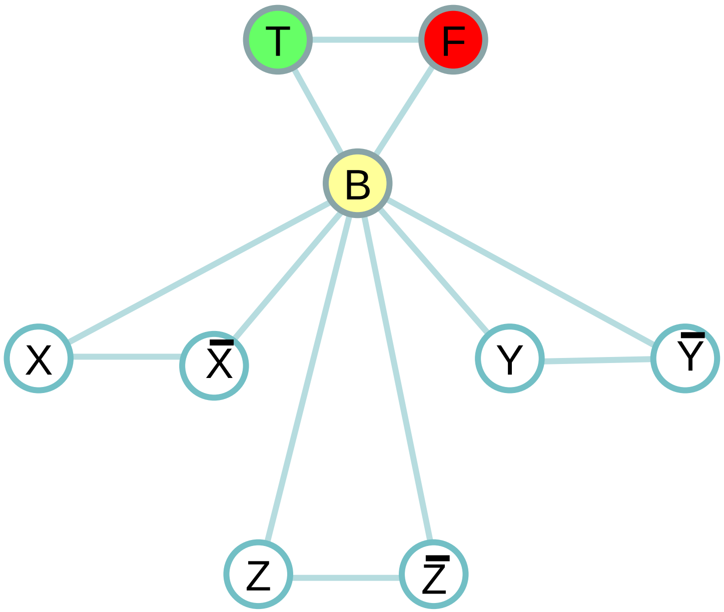
$$3DM <_P XC3$$

Graph Coloring

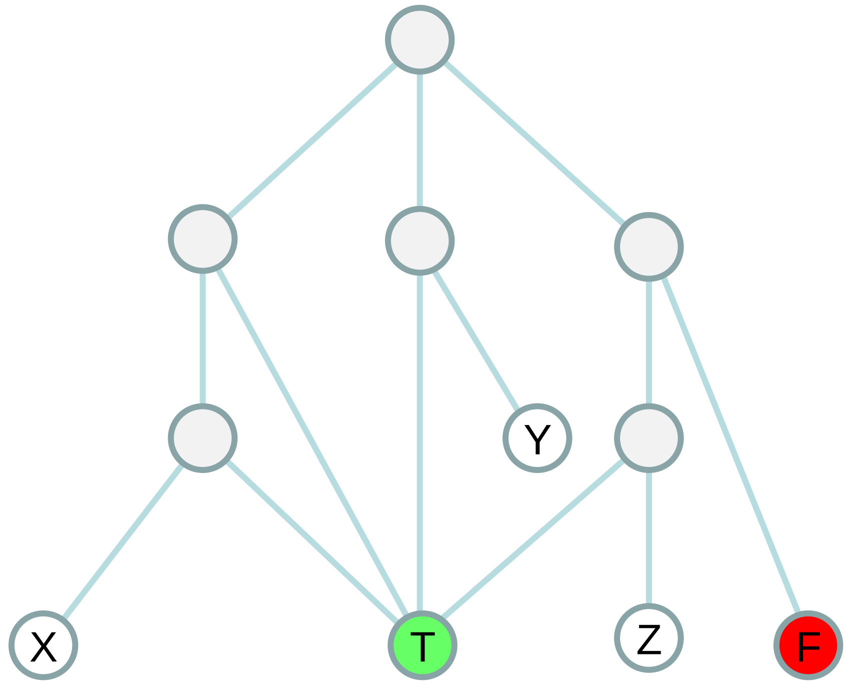
- NP-Complete
  - Graph K-coloring
  - Graph 3-coloring
- Polynomial
  - Graph 2-Coloring



3-SAT $<_P$ 3 Colorability



Truth Setting Gadget



Clause Testing Gadget

(Can be colored if at least one input is T)

Number Problems

- Subset sum problem
  - Given natural numbers $w_1, \ldots, w_n$ and a target number W, is there a subset that adds up to exactly W?

- Subset sum problem is NP-Complete
- Subset Sum problem can be solved in O(nW) time

XC3 $<_P$ SUBSET SUM

Idea: Represent each set as a bit vector, then interpret the bit vectors as integers. Add them up to get the all one's vector.

$\{x_3, x_5, x_9\}$ => 001010001000

Does there exist a subset that sums to exactly 111111111111?

Annoying detail: What about the carries?

Integer Linear Programming

- Linear Programming – minimize linear function subject to linear constraints
- Integer Linear Programming – require an integer solution
- NP Completeness reduction from 3-SAT

Use 0-1 variables for x_i's

Constraint for clause $x_1 \vee \overline{x_2} \vee \overline{x_3}$

$x_1 + (1 - x_2) + (1 - x_3) > 0$

Scheduling with release times and deadlines

- Tasks $T_1,\ldots,T_n$ with release time r_i, deadline d_i, and work w_i
- Reduce from Subset Sum
  - Given natural numbers $w_1, \ldots, w_n$ and a target number K, is there a subset that adds up to exactly K?
  - Suppose the sum $w_1+\ldots+w_n = W$
- Task T_i has release time 0 and deadline W+1
- Add an additional task with release time K, deadline K+1 and work 1