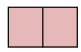
CSE 421
Algorithms

Autumn 2016
Lecture 9
Dijkstra's algorithm

Last Week – Greedy Algorithms

- Task scheduling to minimize maximum lateness
  - Interchange lemma

- Farthest in the future algorithm for optimal caching
  - Discard element whose first occurrence is last in the sequence



A, B, C, A, C, D, C, B, C, A, D

Announcement

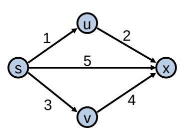
- Collaboration Policy
  - Discussing problems with other students is okay
  - Write ups must be done independently
  - Acknowledge people you work with

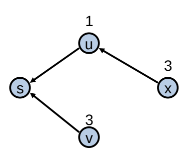
This week

- Topics
  - Dijkstra's Algorithm (Section 4.4)
  - Wednesday: Shortest Paths / Minimum Spanning Trees
  - Friday: Minimum Spanning Trees
- Reading
  - 4.4, 4.5, 4.7, 4.8

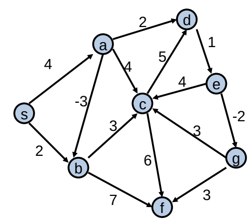
Single Source Shortest Path Problem

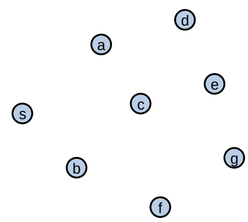
- Given a graph and a start vertex s
  - Determine distance of every vertex from s
  - Identify shortest paths to each vertex
    - Express concisely as a "shortest paths tree"
    - Each vertex has a pointer to a predecessor on shortest path





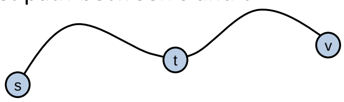
Construct Shortest Path Tree from s





Warmup

- If P is a shortest path from s to v, and if t is on the path P, the segment from s to t is a shortest path between s and t

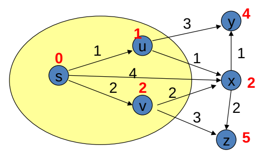


- WHY?

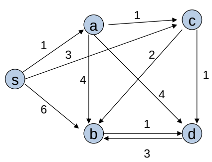
Assume all edges have non-negative cost

Dijkstra's Algorithm

```
S = {};    d[s] = 0;    d[v] = infinity for v != s
While S != V
        Choose v in V-S with minimum d[v]
        Add v to S
        For each  w in the neighborhood of v
                d[w] = min(d[w], d[v] + c(v, w))
```



Simulate Dijkstra's algorithm (starting from s) on the graph



Round	Vertex Added	s	a	b	c	d
1						
2						
3						
4						
5						

Who was Dijkstra?

- What were his major contributions?

http://www.cs.utexas.edu/users/EWD/

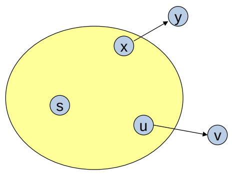
- Edsger Wybe Dijkstra was one of the most influential members of computing science's founding generation. Among the domains in which his scientific contributions are fundamental are
  - algorithm design
  - programming languages
  - program design
  - operating systems
  - distributed processing
  - formal specification and verification
  - design of mathematical arguments

Dijkstra's Algorithm as a greedy algorithm

- Elements committed to the solution by order of minimum distance

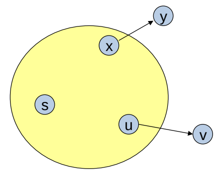
Correctness Proof

- Elements in S have the correct label
- Key to proof: when v is added to S, it has the correct distance label.



Proof

- Let v be a vertex in V-S with minimum d[v]
- Let P_v be a path of length d[v], with an edge (u,v)
- Let P be some other path to v. Suppose P first leaves S on the edge (x, y)
  - $P = P_{sx} + c(x,y) + P_{yv}$
  - $Len(P_{sx}) + c(x,y) >= d[y]$
  - $Len(P_{yv}) >= 0$
  - $Len(P) >= d[y] + 0 >= d[v]$

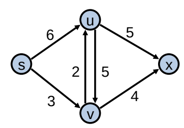


Negative Cost Edges

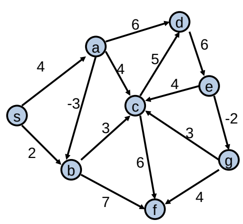
- Draw a small example a negative cost edge and show that Dijkstra's algorithm fails on this example

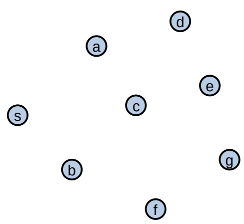
Bottleneck Shortest Path

- Define the bottleneck distance for a path to be the maximum cost edge along the path



Compute the bottleneck shortest paths





How do you adapt Dijkstra's algorithm to handle bottleneck distances

- Does the correctness proof still apply?