

CSE 421

Algorithms

Autumn 2016

Lecture 9

Dijkstra's algorithm

Last Week – Greedy Algorithms

- Task scheduling to minimize maximum lateness
 - Interchange lemma



- Farthest in the future algorithm for optimal caching
 - Discard element whose first occurrence is last in the sequence



A, B, C, A, C, D, C, B, C, A, D

Announcement

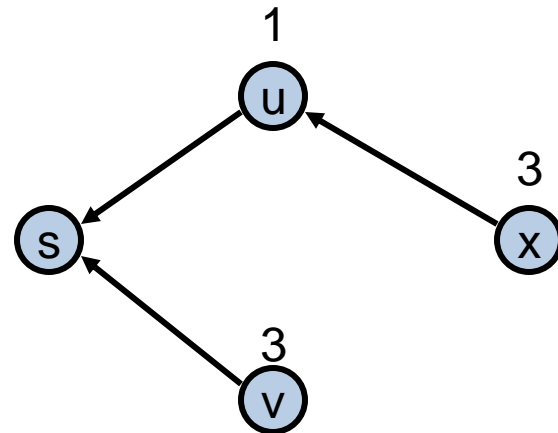
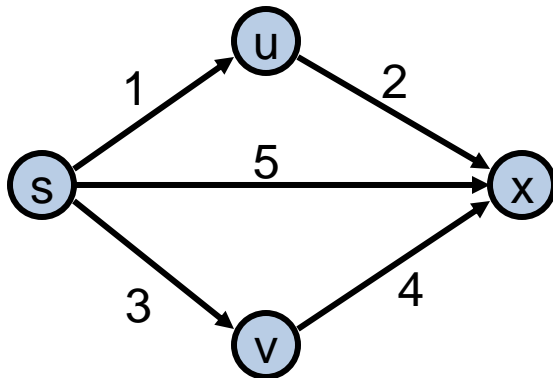
- Collaboration Policy
 - Discussing problems with other students is okay
 - Write ups must be done independently
 - Acknowledge people you work with

This week

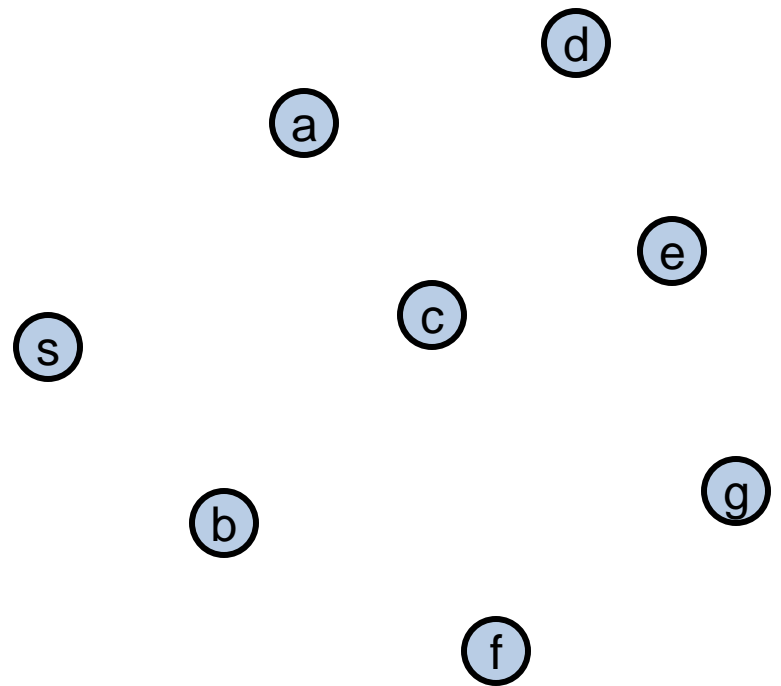
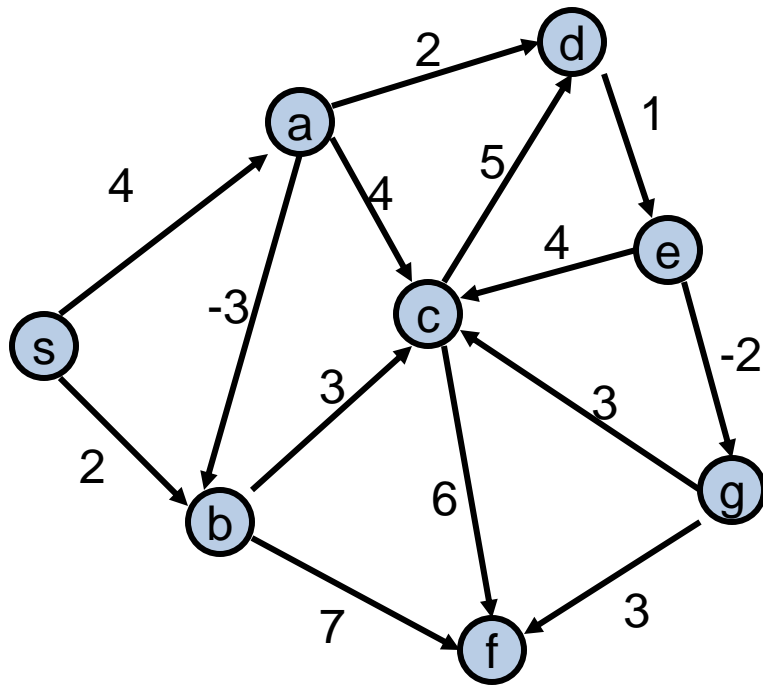
- Topics
 - Dijkstra's Algorithm (Section 4.4)
 - Wednesday: Shortest Paths / Minimum Spanning Trees
 - Friday: Minimum Spanning Trees
- Reading
 - 4.4, 4.5, 4.7, 4.8

Single Source Shortest Path Problem

- Given a graph and a start vertex s
 - Determine distance of every vertex from s
 - Identify shortest paths to each vertex
 - Express concisely as a “shortest paths tree”
 - Each vertex has a pointer to a predecessor on shortest path

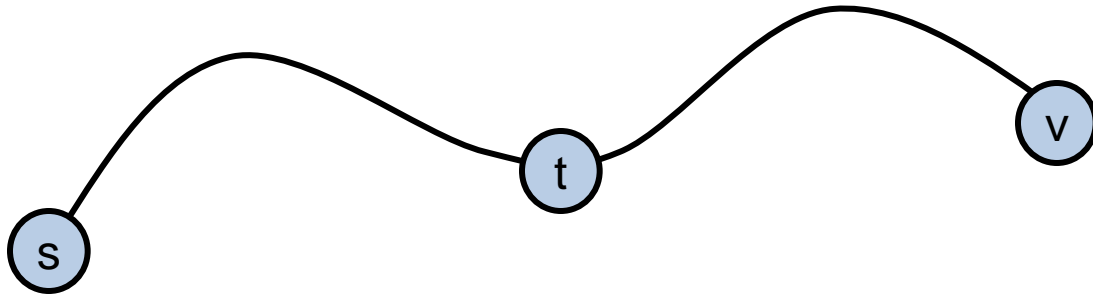


Construct Shortest Path Tree from s



Warmup

- If P is a shortest path from s to v , and if t is on the path P , the segment from s to t is a shortest path between s and t



- WHY?

Assume all edges have non-negative cost

Dijkstra's Algorithm

$S = \{ \}$; $d[s] = 0$; $d[v] = \text{infinity}$ for $v \neq s$

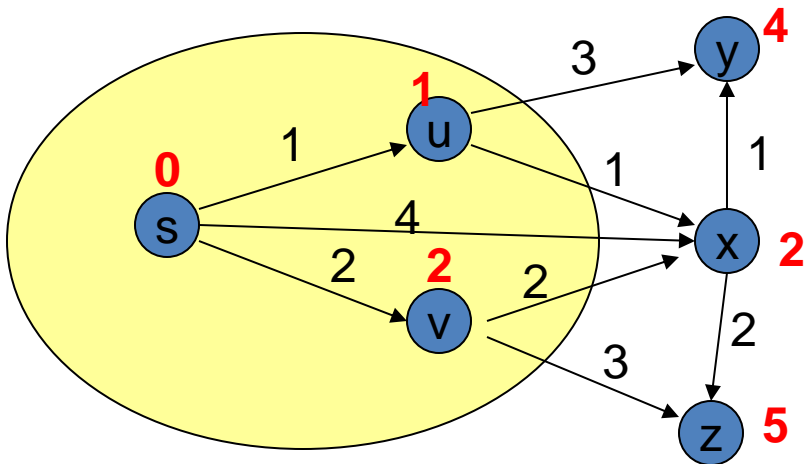
While $S \neq V$

Choose v in $V-S$ with minimum $d[v]$

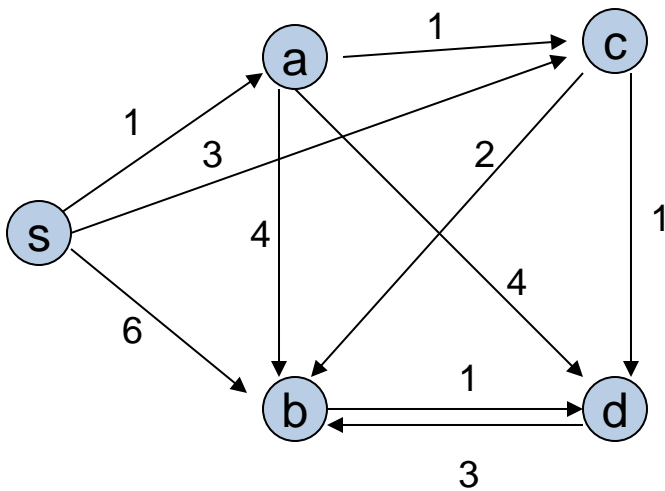
Add v to S

For each w in the neighborhood of v

$$d[w] = \min(d[w], d[v] + c(v, w))$$



Simulate Dijkstra's algorithm (starting from s) on the graph



Round	Vertex Added	s	a	b	c	d
1						
2						
3						
4						
5						

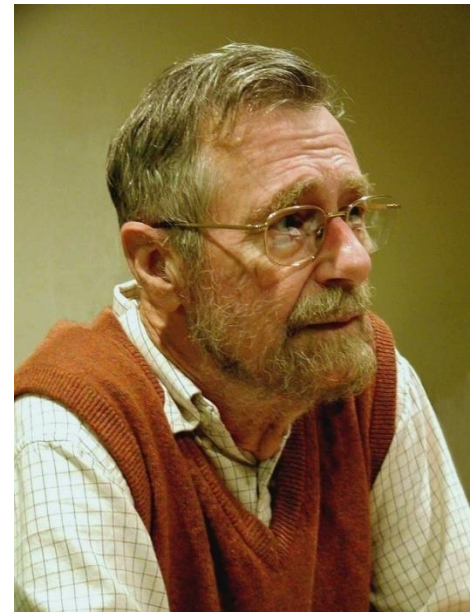
Who was Dijkstra?

- What were his major contributions?



<http://www.cs.utexas.edu/users/EWD/>

- Edsger Wybe Dijkstra was one of the most influential members of computing science's founding generation. Among the domains in which his scientific contributions are fundamental are
 - algorithm design
 - programming languages
 - program design
 - operating systems
 - distributed processing
 - formal specification and verification
 - design of mathematical arguments

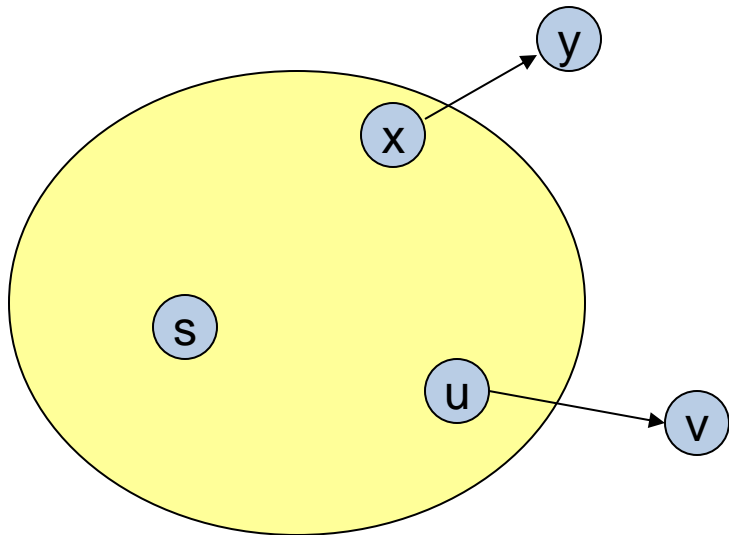


Dijkstra's Algorithm as a greedy algorithm

- Elements committed to the solution by order of minimum distance

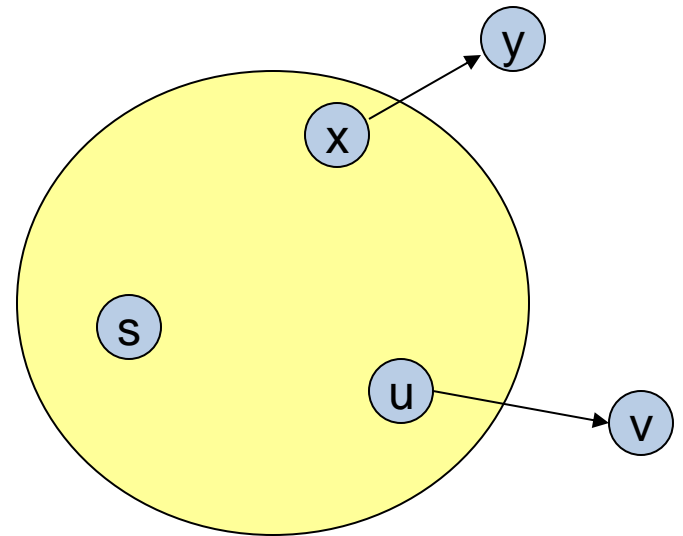
Correctness Proof

- Elements in S have the correct label
- Key to proof: when v is added to S , it has the correct distance label.



Proof

- Let v be a vertex in $V-S$ with minimum $d[v]$
- Let P_v be a path of length $d[v]$, with an edge (u,v)
- Let P be some other path to v . Suppose P first leaves S on the edge (x, y)
 - $P = P_{sx} + c(x,y) + P_{yv}$
 - $\text{Len}(P_{sx}) + c(x,y) \geq d[y]$
 - $\text{Len}(P_{yv}) \geq 0$
 - $\text{Len}(P) \geq d[y] + 0 \geq d[v]$

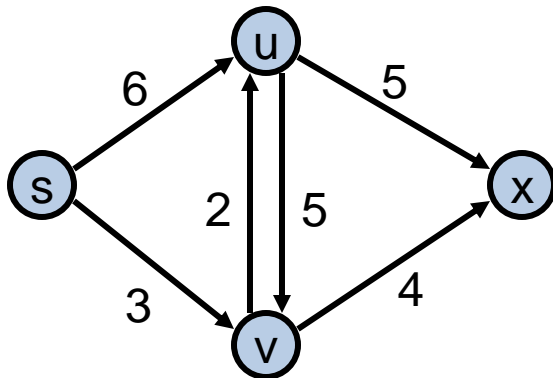


Negative Cost Edges

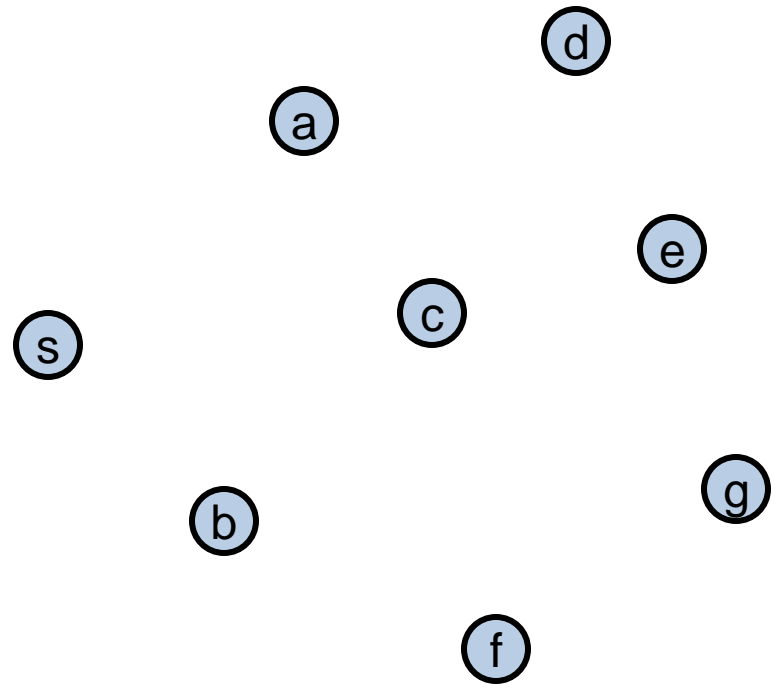
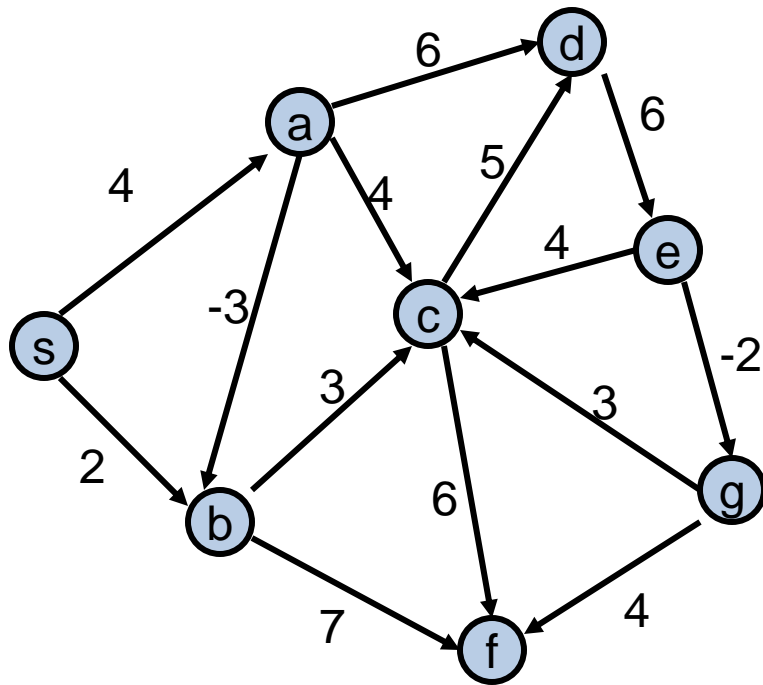
- Draw a small example a negative cost edge and show that Dijkstra's algorithm fails on this example

Bottleneck Shortest Path

- Define the bottleneck distance for a path to be the maximum cost edge along the path



Compute the bottleneck shortest paths



How do you adapt Dijkstra's algorithm to handle bottleneck distances

- Does the correctness proof still apply?