# CSE 421
# Algorithms

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Lecture 4

## Announcements

- Reading
  - Chapter 2.1, 2.2
  - Chapter 3 (Mostly review)
  - Start on Chapter 4
- Homework Guidelines
  - Prove that your algorithm works
    - A proof is a "convincing argument"
  - Give the run time for you algorithm
    - Justify that the algorithm satisfies the runtime bound
  - You may lose points for style

## What does it mean for an algorithm to be efficient?

## Definitions of efficiency

- Fast in practice
- Qualitatively better worst case performance than a brute force algorithm

## Polynomial time efficiency

- An algorithm is efficient if it has a polynomial run time
- Run time as a function of problem size
  - Run time: count number of instructions executed on an underlying model of computation
  - T(n): maximum run time for all problems of size at most n

## Polynomial Time

- Algorithms with polynomial run time have the property that increasing the problem size by a constant factor increases the run time by at most a constant factor (depending on the algorithm)

## Why Polynomial Time?

- Generally, polynomial time seems to capture the algorithms which are efficient in practice
- The class of polynomial time algorithms has many good, mathematical properties

## Polynomial vs. Exponential Complexity

- Suppose you have an algorithm which takes n! steps on a problem of size n
- If the algorithm takes one second for a problem of size 10, estimate the run time for the following problems sizes:

12 14 16 18 20

## Ignoring constant factors

- Express run time as O(f(n))
- Emphasize algorithms with slower growth rates
- Fundamental idea in the study of algorithms
- Basis of Tarjan/Hopcroft Turing Award

## Why ignore constant factors?

- Constant factors are arbitrary
  - Depend on the implementation
  - Depend on the details of the model
- Determining the constant factors is tedious and provides little insight

## Why emphasize growth rates?

- The algorithm with the lower growth rate will be faster for all but a finite number of cases
- Performance is most important for larger problem size
- As memory prices continue to fall, bigger problem sizes become feasible
- Improving growth rate often requires new techniques

## Formalizing growth rates

- T(n) is O(f(n)) $[T : Z^+ \rightarrow R^+]$
  - If n is sufficiently large, T(n) is bounded by a constant multiple of f(n)
  - Exist c, $n_0$, such that for $n > n_0$, $T(n) < c\, f(n)$
- T(n) is O(f(n)) will be written as: T(n) = O(f(n))
  - Be careful with this notation

## Prove $3n^2 + 5n + 20$ is $O(n^2)$

Let c =

Let $n_0$ =

T(n) is O(f(n)) if there exist c, $n_0$, such that for $n > n_0$, $T(n) < c\, f(n)$

## Order the following functions in increasing order by their growth rate

a) $n \log^4 n$
b) $2n^2 + 10n$
c) $2^{n/100}$
d) $1000n + \log^8 n$
e) $n^{100}$
f) $3^n$
g) $1000 \log^{10} n$
h) $n^{1/2}$

## Lower bounds

- T(n) is $\Omega(f(n))$
  - T(n) is at least a constant multiple of f(n)
  - There exists an $n_0$, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$
- Warning: definitions of $\Omega$ vary
- T(n) is $\Theta(f(n))$ if T(n) is O(f(n)) and T(n) is $\Omega(f(n))$

## Useful Theorems

- If lim (f(n) / g(n)) = c for c > 0 then $f(n) = \Theta(g(n))$
- If f(n) is O(g(n)) and g(n) is O(h(n)) then f(n) is O(h(n))
- If f(n) is O(h(n)) and g(n) is O(h(n)) then f(n) + g(n) is O(h(n))

## Ordering growth rates

- For $b > 1$ and $x > 0$
  - $\log^b n$ is $O(n^x)$
- For $r > 1$ and $d > 0$
  - $n^d$ is $O(r^n)$

## Graph Theory

- G = (V, E)
  - V – vertices
  - E – edges
- Undirected graphs
  - Edges sets of two vertices {u, v}
- Directed graphs
  - Edges ordered pairs (u, v)
- Many other flavors
  - Edge / vertices weights
  - Parallel edges
  - Self loops

# Definitions

- Path: $v_1, v_2, \ldots, v_k$, with $(v_i, v_{i+1})$ in E
  - Simple Path
  - Cycle
  - Simple Cycle
- Neighborhood
  - N(v)
- Distance
- Connectivity
  - Undirected
  - Directed (strong connectivity)
- Trees
  - Rooted
  - Unrooted

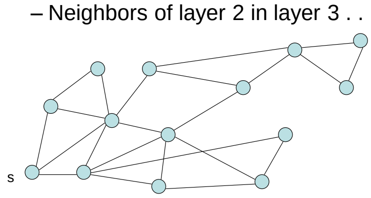
# Graph search

- Find a path from s to t

```
S = {s}
while S is not empty
        u = Select(S)
        visit u
        foreach v in N(u)
                if v is unvisited
                        Add(S, v)
                        Pred[v] = u
                if (v = t) then path found
```

# Breadth first search

- Explore vertices in layers
  - s in layer 1
  - Neighbors of s in layer 2
  - Neighbors of layer 2 in layer 3 . . .




# Key observation

- All edges go between vertices on the same layer or adjacent layers