CSE 421
Algorithms

Richard Anderson

Autumn 2016

Lecture 1

CSE 421 Course Introduction

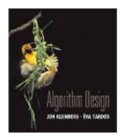
- CSE 421, Introduction to Algorithms
  - MWF, 1:30-2:20 pm
  - MGH 241
- Instructor
  - Richard Anderson, anderson@cs.washington.edu
  - Office hours:
    - CSE 582
    - Office hours: Monday 2:30-3:30, Wednesday 2:30-3:30
- Teaching Assistants
  - Deepali Aneja
  - Maxwell Horton
  - Benjamin Jones

Announcements

- It's on the web.
- Homework due Wednesdays
  - HW 1, Due October 5, 2015
  - It's on the web (or will be soon)
- You should be on the course mailing list
  - But it will probably go to your uw.edu account

Text book

- Algorithm Design
- Jon Kleinberg, Eva Tardos
- Read Chapters 1 & 2
- Expected coverage:
  - Chapter 1 through 7
- Book available at:
  - UW Bookstore ($163.50)
  - Ebay ($25.30)
  - Amazon ($19.79 and up)
  - Kindle ($104.99)
  - PDF



Course Mechanics

- Homework
  - Due Wednesdays
  - About 5 problems, sometimes programming
  - Target: 1 week turnaround on grading
- Exams (In class)
  - Midterm, Monday, October 31 (probably)
  - Final, Monday, December 12, 2:30-4:20 pm
- Approximate grade weighting
  - HW: 50, MT: 15, Final: 35
- Course web
  - Slides, Handouts

All of Computer Science is the Study of Algorithms

How to study algorithms

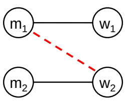
- Zoology
- Mine is faster than yours is
- Algorithmic ideas
  - Where algorithms apply
  - What makes an algorithm work
  - Algorithmic thinking

Introductory Problem: Stable Matching

- Setting:
  - Assign TAs to Instructors
  - Avoid having TAs and Instructors wanting changes
    - E.g., Prof A. would rather have student X than her current TA, and student X would rather work for Prof A. than his current instructor.

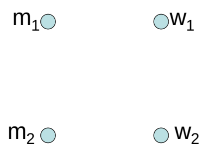
Formal notions

- Perfect matching
- Ranked preference lists
- Stability



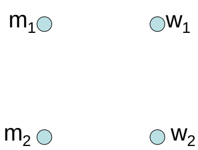
Example (1 of 3)

m_1: w_1 w_2
m_2: w_2 w_1
w_1: m_1 m_2
w_2: m_2 m_1



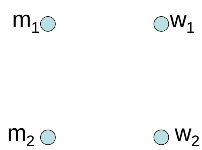
Example (2 of 3)

m_1: w_1 w_2
m_2: w_1 w_2
w_1: m_1 m_2
w_2: m_1 m_2



Example (3 of 3)

m_1: w_1 w_2
m_2: w_2 w_1
w_1: m_2 m_1
w_2: m_1 m_2



Formal Problem

- Input
  - Preference lists for $m_1, m_2, \ldots, m_n$
  - Preference lists for $w_1, w_2, \ldots, w_n$
- Output
  - Perfect matching M satisfying stability property:

> If (m', w') ∈ M and (m'', w'') ∈ M then
> (m' prefers w' to w'') or (w'' prefers m'' to m')

Idea for an Algorithm

m proposes to w

If w is unmatched, w accepts

If w is matched to m_2

If w prefers m to m_2 w accepts m, dumping m_2

If w prefers m_2 to m, w rejects m

Unmatched m proposes to the highest w on its preference list that it has not already proposed to

Algorithm

```
Initially all m in M and w in W are free
While there is a free m
        w highest on m's list that m has not proposed to
        if w is free, then match (m, w)
        else
                suppose (m2, w) is matched
                if w prefers m to m2
                        unmatch (m2, w)
                        match (m, w)
```

Example

m_1: w_1 w_2 w_3

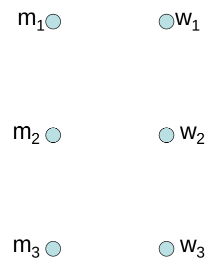
m_2: w_1 w_3 w_2

m_3: w_1 w_2 w_3

w_1: m_2 m_3 m_1

w_2: m_3 m_1 m_2

w_3: m_3 m_1 m_2



Does this work?

- Does it terminate?
- Is the result a stable matching?

- Begin by identifying invariants and measures of progress
  - m's proposals get worse (have higher m-rank)
  - Once w is matched, w stays matched
  - w's partners get better (have lower w-rank)

Claim: If an m reaches the end of its list, then all the w's are matched

Claim: The algorithm stops in at most n^2 steps

When the algorithms halts, every w is matched

Why?

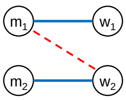
Hence, the algorithm finds a perfect matching

The resulting matching is stable

Suppose

$(m_1, w_1) \in M$, $(m_2, w_2) \in M$

m_1 prefers w_2 to w_1



How could this happen?

Result

- Simple, $O(n^2)$ algorithm to compute a stable matching
- Corollary
  - A stable matching always exists