University of Washington
Department of Computer Science and Engineering
CSE 421, Autumn 2016

October 11, 2016

Homework 3, Due Wednesday October 19, 2016

Problem 1 (10 points):

Page 110, Exercise 9.

Problem 2 (10 points):

Page 110, Exercise 10.

Problem 3 (10 points):

Consider a directed graph on n vertices, where each vertex has exactly one outgoing edge. This graph consists of a collection of cycles as well as additional vertices that have paths to the cycles, which we call the *branches*. We define the *weight* of the cycle to be the total number of vertices that are either on the cycle or on branches that are connected to the cycle.

Describe a linear time algorithm that identifies all of the cycles and computes the length and weight of each cycle.

Programming Problem 4 (20 points) :

(10 points) Implement your algorithm for finding the cycles in an out-degree one graph. Your algorithm should be designed to work on very large graphs, e.g., with $n = 100,000,000$.

Write an input generator which creates completely random out-degree one graphs where each vertex points to another vertex chosen uniformly at random. The graph is allowed to have self-loops.

As the size of the problem increases - how does the number of cycles, and the length, and weight of the cycles change, when the input is a random graph with out-degree one? Give a table of results. Do the results match your intuition? Do you have any conjectures on the growth rates of number of cycles, cycle length, and cycle weight?

You are free to write in any programming language you like. The quality of your algorithm may be graded, but the actual quality of the code will not be graded. The expectation is that you will write the algorithmic code yourself - but you can use other code or libraries for supporting operations.

Problem 5 (10 points):

Page 190, Exercise 5. Justify that your algorithm is correct.

Problem 6 (10 points):

Page 191, Exercise 7. Justify that your algorithm is correct.