University of Washington
Department of Computer Science and Engineering
CSE 421, Autumn 2015

November 1, 2015

Midterm Exam, Monday, November 2, 2015

NAME: ____________________

Instructions:

- Closed book, closed notes, no calculators
- Time limit: 50 minutes
- Answer the problems on the exam paper.
- If you need extra space use the back of a page
- Problems are not of equal difficulty, if you get stuck on a problem, move on.

1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
Total	/60

Problem 1 (10 points):

Show that the Gale-Shipley Stable Marriage algorithm can take $\Theta(n^2)$ steps with appropriate choice of preference lists. Give preference lists and an ordering of the proposals that requires $\Theta(n^2)$ steps. Explain why your example achieves the bound.

Hint: this can be done with all of the M's having the same preference lists, and all of the W's having the same preference lists.

Problem 2 (10 points):

Let p, q, and r be positive constants. Prove that $pn^2 + qn + r$ is $O(n^2)$ using the formal definition of $O(\cdot)$.

Big $O(\cdot)$ definition: $f(n)$ is $O(g(n))$ if there exists $c > 0$ and $n_0 \geq 0$ such that for all $n \geq n_0$, $f(n) \leq cg(n)$.

Problem 3 (10 points):

a) Let $G = (V, E)$ be a directed graph, with v a vertex in V. Give an $O(n + m)$ time algorithm to find the shortest cycle in G that contains v.

b) Let $G = (V, E)$ by an $x \times y$ grid graph, where

$$V = \{(i, j) \mid 0 \leq i < x \text{ and } 0 \leq j < y\}$$

and

$$E = \{((i, j), (i{+}1, j)) \mid 0 \leq i < x{-}1 \text{ and } 0 \leq j < y\} \cup \{((i, j), (i, j{+}1)) \mid 0 \leq i < x \text{ and } 0 \leq j < y{-}1\}$$

Is G a bipartite graph? Justify your answer.

Problem 4 (10 points):

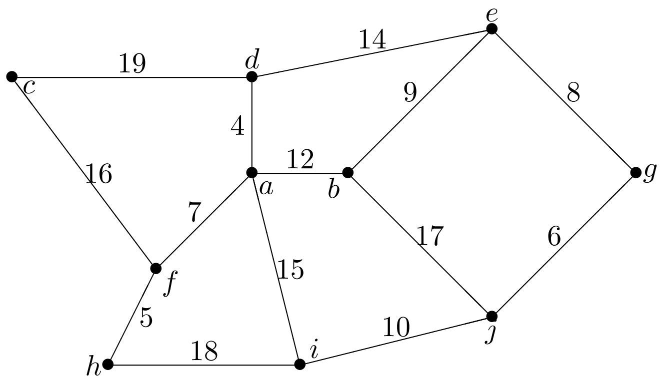
The input for an interval scheduling problem is a set of intervals $I = \{i_1, \ldots, i_n\}$ where i_k has start time s_k, and finish time f_k. The problem is to find a set of non-overlapping intervals that satisfies a given criteria.

a) Suppose that you want to maximize the total length of the selected intervals. **True or false**: The greedy algorithm based on selecting intervals in order of decreasing length finds an optimal solution. Justify your answer.

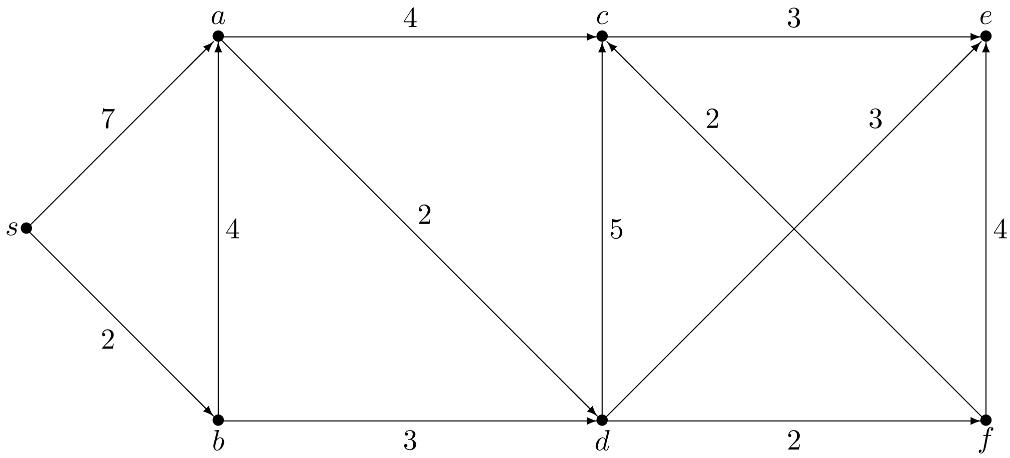
b) The set of intervals $I' = \{i'_1, \ldots, i'_n\}$ is said to be a *shrinking* of the intervals $I'' = \{i''_1, \ldots, i''_n\}$ if each interval in I' is contained in the corresponding interval of I'', in other words, for $1 \leq k \leq n$, $s''_k \leq s'_k \leq f'_k \leq f''_k$. **True or false**: If I' is a shrinking of I'', then the maximum number of non-overlapping intervals in I' is at least as great as the maximum number of non-overlapping intervals in I''. Justify your answer.

Problem 5 (10 points):

a) Construct a minimum spanning tree for the following graph using Prim's algorithm starting from vertex a. Indicate the order that vertices are added to the graph.



b) Solve the single source shortest path problem in the following graph starting from s using Dijkstra's algorithm. Indicate the order that vertices are added to S, and their value when they are added to S.



Problem 6 (10 points):

Consider the following recurrence:

$$T(n) = \begin{cases} T(\frac{n}{2}) + T(\frac{n}{3}) + n & \text{if } n \geq 1 \\ 0 & \text{if } n < 1 \end{cases}$$

a) Unroll the recurrence to determine the first four levels of the recursion tree. (This means expanding three times, as the first level is the root, corresponding to the problem of size n.)

b) What is the minimum and maximum depth of leaves in the recursion tree? (The leaves are the nodes that correspond to a problem size of 1.)

c) Show that the solution to the recurrence satisfies $T(n) \leq 6n$.