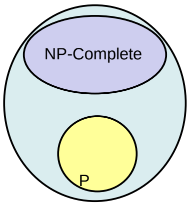


CSE 421 Algorithms

Richard Anderson

Lecture 29

Complexity Theory

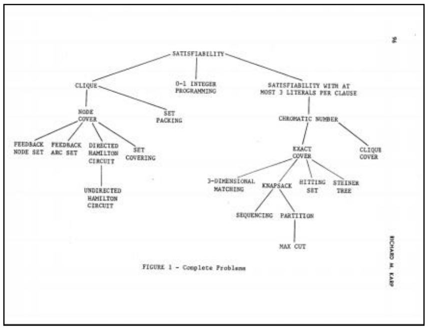
Announcements

- Final exam,
  - Monday, December 14, 2:30-4:20 pm
  - Comprehensive (2/3 post midterm, 1/3 pre midterm)
- Review session
  - Friday, 3:30 – 5:00 pm. More 220
- Online course evaluations available

NP Complete Problems

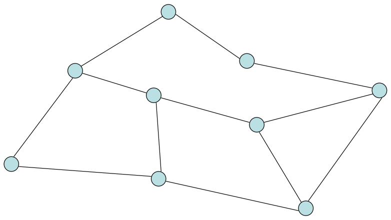
1. Circuit Satisfiability
2. Formula Satisfiability
   a. 3-SAT
3. Graph Problems
   a. Independent Set
   b. Vertex Cover
   c. Clique
4. Path Problems
   a. Hamiltonian cycle
   b. Hamiltonian path
5. Partition Problems
   a. Three dimensional matching
   b. Exact cover
6. Graph Coloring
7. Number problems
   a. Subset sum
8. Integer linear programming
9. Scheduling with release times and deadlines

Karp's 21 NP Complete Problems

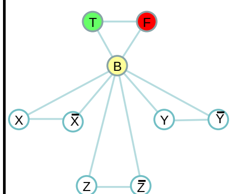


A final NP completeness result: Graph Coloring

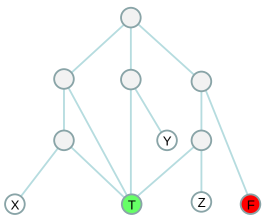
- NP-Complete
  - Graph K-coloring
  - Graph 3-coloring
- Polynomial
  - Graph 2-Coloring



3-SAT $<_P$ 3 Colorability



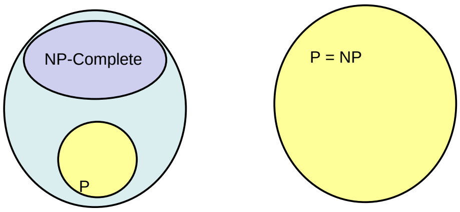
Truth Setting Gadget



Clause Testing Gadget

What we don't know

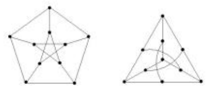
- P vs. NP



If P != NP, is there anything in between

- Yes, Ladner [1975]
- Problems not known to be in P or NP Complete
  - Factorization
  - Discrete Log
  - Graph Isomorphism

Solve $g^k = b$ over a finite group

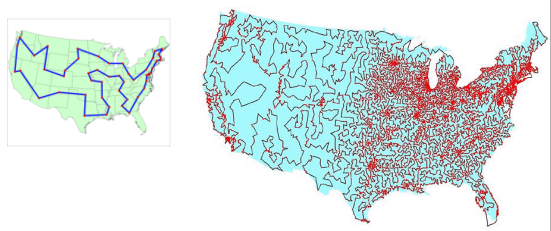


Coping with NP Completeness

- Approximation Algorithms
  - Christofides algorithm for TSP (Undirected graphs satisfying triangle inequality)
- Solution guarantees on greedy algorithms
  - Bin packing

Coping with NP-Completeness

- Branch and Bound
  - Euclidean TSP

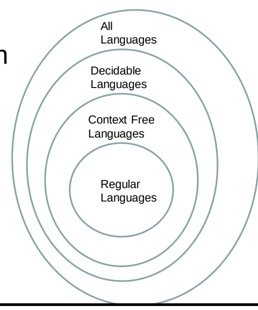


Coping with NP-Completeness

- Local Search
  - Modify solution until a local minimum is reached
    - Interchange algorithm for TSP
    - Recoloring algorithms
  - Simulated annealing

Complexity Theory

- Computational requirements to recognize languages
- Models of Computation
- Resources
- Hierarchies



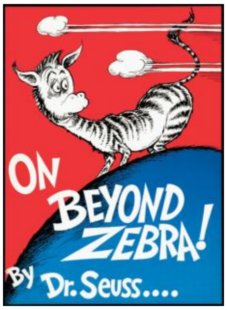
Time complexity

- P: (Deterministic) Polynomial Time
- NP: Non-deterministic Polynomial Time
- EXP: Exponential Time

Space Complexity

- Amount of Space (Exclusive of Input)
- L: Logspace, problems that can be solved in O(log n) space for input of size n
- PSPACE, problems that can be required in a polynomial amount of space

So what is beyond NP?



NP vs. Co-NP

- Given a Boolean formula, is it true for some choice of inputs
- Given a Boolean formula, is it true for all choices of inputs

Problems beyond NP

- Exact TSP, Given a graph with edge lengths and an integer K, does the minimum tour have length K
- Minimum circuit, Given a circuit C, is it true that there is no smaller circuit that computes the same function a C

Polynomial Hierarchy

- Level 1
  - $\exists X_1\ \Phi(X_1)$, $\forall X_1\ \Phi(X_1)$
- Level 2
  - $\forall X_1 \exists X_2\ \Phi(X_1,X_2)$, $\exists X_1 \forall X_2\ \Phi(X_1,X_2)$
- Level 3
  - $\forall X_1 \exists X_2 \forall X_3\ \Phi(X_1,X_2,X_3)$, $\exists X_1 \forall X_2 \exists X_3\ \Phi(X_1,X_2,X_3)$

Polynomial Space

- Quantified Boolean Expressions
  - $\exists X_1 \forall X_2 \exists X_3 \ldots \exists X_{n-1} \forall X_n \; \Phi(X_1, X_2, X_3 \ldots X_{n-1} X_n)$
- Space bounded games
  - Competitive Facility Location Problem
- Counting problems
  - The number of Hamiltonian Circuits in a graph