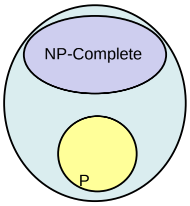


CSE 421 Algorithms

Richard Anderson

Lecture 28

Survey of NP Complete Problems

Announcements

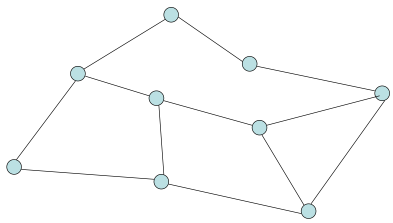
- Final exam,
  - Monday, December 14, 2:30-4:20 pm
  - Comprehensive (2/3 post midterm, 1/3 pre midterm)
- Review session
  - Friday, 3:30 – 5:00 pm. More 220
- Online course evaluations available

NP Complete Problems

1. Circuit Satisfiability
2. Formula Satisfiability
   a. 3-SAT
3. Graph Problems
   a. Independent Set
   b. Vertex Cover
   c. Clique
4. Path Problems
   a. Hamiltonian cycle
   b. Hamiltonian path
5. Partition Problems
   a. Three dimensional matching
   b. Exact cover
6. Graph Coloring
7. Number problems
   a. Subset sum
8. Integer linear programming
9. Scheduling with release times and deadlines

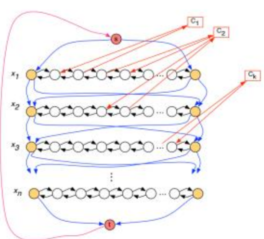
Hamiltonian Circuit Problem

- Hamiltonian Circuit – a simple cycle including all the vertices of the graph



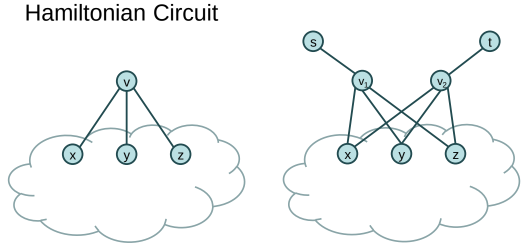
Thm: Hamiltonian Circuit is NP Complete

- Reduction from 3-SAT



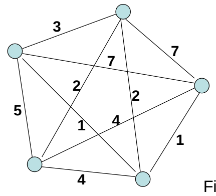
Reduce Hamiltonian Circuit to Hamiltonian Path

G_2 has a Hamiltonian Path iff G_1 has a Hamiltonian Circuit



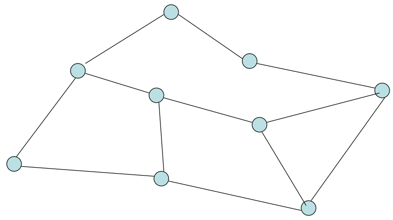
Traveling Salesman Problem

- Given a complete graph with edge weights, determine the shortest tour that includes all of the vertices (visit each vertex exactly once, and get back to the starting point)

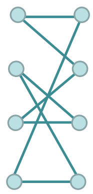


Find the minimum cost tour

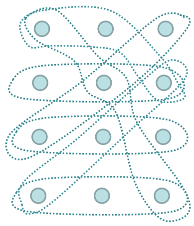
Thm: HC $<_P$ TSP



Matching

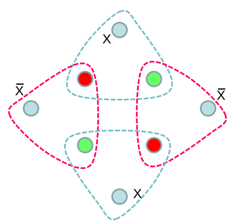


Two dimensional matching

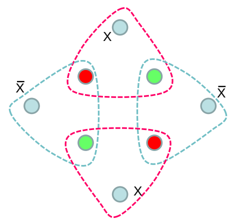


Three dimensional matching (3DM)

3-SAT $<_P$ 3DM



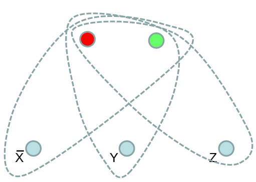
X True



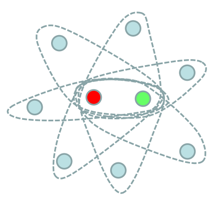
X False

Truth Setting Gadget

3-SAT $<_P$ 3DM



Clause gadget for ($\bar{X}$ OR Y OR Z)



Garbage Collection Gadget
(Many copies)

Exact Cover (sets of size 3) XC3

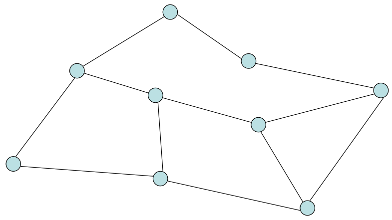
Given a collection of sets of size 3 of a domain of size 3N, is there a sub-collection of N sets that cover the sets

(A, B, C), (D, E, F), (A, B, G),
(A, C, I), (B, E, G), (A, G, I),
(B, D, F), (C, E, I), (C, D, H),
(D, G, I), (D, F, H), (E, H, I),
(F, G, H), (F, H, I)

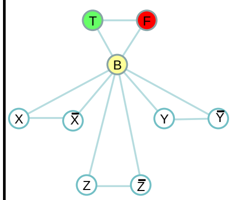
3DM $<_P$ XC3

Graph Coloring

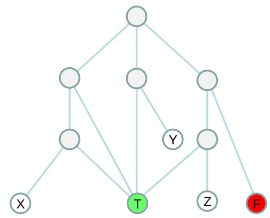
- NP-Complete
  - Graph K-coloring
  - Graph 3-coloring
- Polynomial
  - Graph 2-Coloring



3-SAT $<_P$ 3 Colorability



Truth Setting Gadget



Clause Testing Gadget

Number Problems

- Subset sum problem
  - Given natural numbers $w_1, \ldots, w_n$ and a target number W, is there a subset that adds up to exactly W?
- Subset sum problem is NP-Complete
- Subset Sum problem can be solved in O(nW) time

XC3 $<_P$ SUBSET SUM

Idea: Represent each set as a bit vector, then interpret the bit vectors as integers. Add them up to get the all one's vector.

{x3, x5, x9} => 001010001000

Does there exist a subset that sums to exactly 111111111111?

Annoying detail: What about the carries?

Integer Linear Programming

Scheduling with release times and deadlines