# CSE 421 Algorithms

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Lecture 14

Divide and Conquer

## Announcements

- Review session, 3:30 pm. CSE 403.
- Midterm. Monday.

## What you really need to know about recurrences

- Work per level changes geometrically with the level
- Geometrically increasing (x > 1)
  - The bottom level wins
- Geometrically decreasing (x < 1)
  - The top level wins
- Balanced (x = 1)
  - Equal contribution

## $T(n) = aT(n/b) + n^c$

- Balanced: $a = b^c$
- Increasing: $a > b^c$
- Decreasing: $a < b^c$

## Divide and Conquer Algorithms

- Split into sub problems
- Recursively solve the problem
- Combine solutions

- Make progress in the split and combine stages
  - Quicksort – progress made at the split step
  - Mergesort – progress made at the combine step
- D&C Algorithms
  - Strassen's Algorithm – Matrix Multiplication
  - Inversions
  - Median
  - Closest Pair
  - Integer Multiplication
  - FFT

## Inversion Problem

- Let $a_1, . . . a_n$ be a permutation of 1 . . n
- $(a_i, a_j)$ is an inversion if $i < j$ and $a_i > a_j$

4, 6, 1, 7, 3, 2, 5

- Problem: given a permutation, count the number of inversions
- This can be done easily in $O(n^2)$ time
  - Can we do better?

## Application

- Counting inversions can be use to measure how close ranked preferences are
  - People rank 20 movies, based on their rankings you cluster people who like that same type of movie

## Counting Inversions

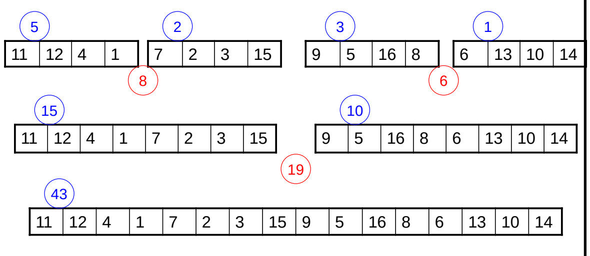
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 12 | 4 | 1 | 7 | 2 | 3 | 15 | 9 | 5 | 16 | 8 | 6 | 13 | 10 | 14 |

Count inversions on lower half

Count inversions on upper half

Count the inversions between the halves

## Count the Inversions




## Problem – how do we count inversions between sub problems in O(n) time?

- Solution – Count inversions while merging

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 7 | 11 | 12 | 15 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5 | 6 | 8 | 9 | 10 | 13 | 14 | 16 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | |

Standard merge algorithm – add to inversion count when an element is moved from the upper array to the solution

## Use the merge algorithm to count inversions

| | | | |
|---|---|---|---|
| 1 | 4 | 11 | 12 |

| | | | |
|---|---|---|---|
| 2 | 3 | 7 | 15 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

| | | | |
|---|---|---|---|
| 5 | 8 | 9 | 16 |

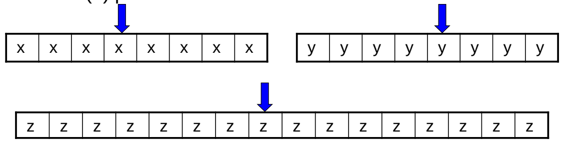
| | | | |
|---|---|---|---|
| 6 | 10 | 13 | 14 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

Indicate the number of inversions for each element detected when merging

## Inversions

- Counting inversions between two sorted lists
  - O(1) per element to count inversions



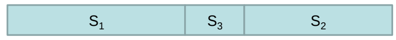

- Algorithm summary
  - Satisfies the "Standard recurrence"
  - T(n) = 2 T(n/2) + cn

# Computing the Median

- Given n numbers, find the number of rank n/2
- Selection, given n numbers and an integer k, find the k-th largest

# Select(A, k)

```
Select(A, k){
        Choose element x from A
        S1 = {y in A | y < x}
        S2 = {y in A | y > x}
        S3 = {y in A | y = x}
        if (|S2| >= k)
                return Select(S2, k)
        else if (|S2| + |S3| >= k)
                return x
        else
                return Select(S1, k - |S2| - |S3|)
}
```




# Randomized Selection

- Choose the element at random
- Analysis can show that the algorithm has expected run time O(n)

# Deterministic Selection

- What is the run time of select if we can guarantee that choose finds an x such that $|S_1| < 3n/4$ and $|S_2| < 3n/4$

# BFPRT Algorithm





- A very clever choose algorithm . . .

Split into n/5 sets of size 5
M be the set of medians of these sets
Let x be the median of M

# BFPRT runtime

$|S_1| < 3n/4$, $|S_2| < 3n/4$

Split into n/5 sets of size 5
M be the set of medians of these sets
x be the median of M
Construct $S_1$ and $S_2$
Recursive call in $S_1$ or $S_2$

## BFPRT Recurrence

- $T(n) <= T(3n/4) + T(n/5) + c\ n$

Prove that $T(n) <= 20\ c\ n$