# CSE 421
# Algorithms

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Lecture 12

Recurrences

# Announcements

- Midterm
  - Monday, Nov 2, in class, closed book
  - Through section 5.2

# Divide and Conquer

- Recurrences, Sections 5.1 and 5.2
- Algorithms
  - Counting Inversions (5.3)
  - Closest Pair (5.4)
  - Multiplication (5.5)
  - FFT (5.6)

# Divide and Conquer

```
Array Mergesort(Array a){
        n = a.Length;
        if (n <= 1)
                return a;
        b = Mergesort(a[0 .. n/2]);
        c = Mergesort(a[n/2+1 .. n-1]);
        return Merge(b, c);
}
```

# Algorithm Analysis

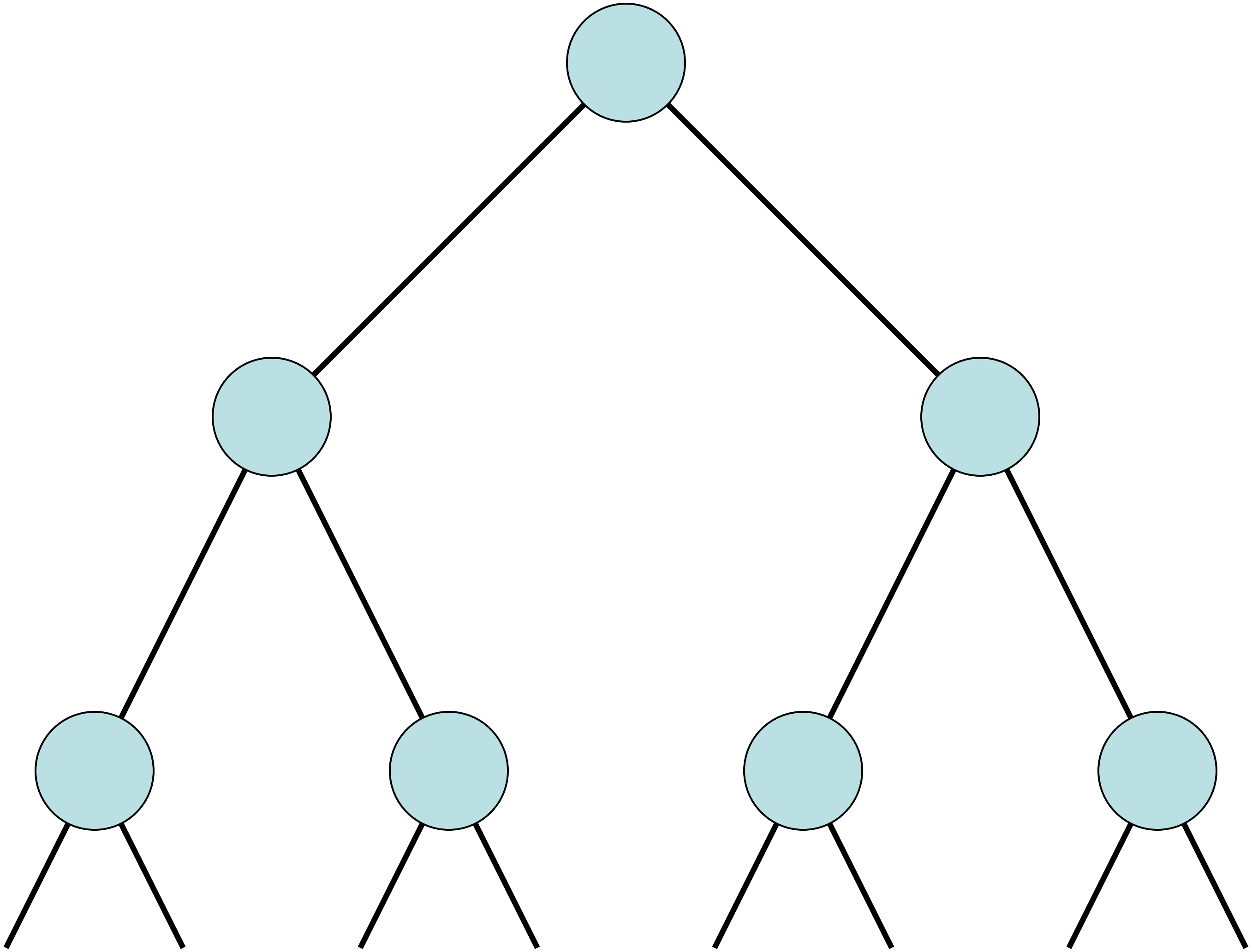
- Cost of Merge
- Cost of Mergesort

$$T(n) <= 2T(n/2) + cn; \; T(1) <= c;$$

# Recurrence Analysis

- Solution methods
  - Unrolling recurrence
  - Guess and verify
  - Plugging in to a “Master Theorem”

# Unrolling the recurrence



# Substitution

Prove $T(n) <= cn (\log_2 n + 1)$ for $n >= 1$

Induction:
Base Case:

Induction Hypothesis:

# A better mergesort (?)

- Divide into 3 subarrays and recursively sort
- Apply 3-way merge

What is the recurrence?

# Unroll recurrence for $T(n) = 3T(n/3) + dn$

$$T(n) = aT(n/b) + f(n)$$

$$T(n) = T(n/2) + cn$$

Where does this recurrence arise?

# Solving the recurrence exactly

$$T(n) = 4T(n/2) + cn$$

$$T(n) = 2T(n/2) + n^2$$

$$T(n) = 2T(n/2) + n^{1/2}$$

# Recurrences

- Three basic behaviors
  - Dominated by initial case
  - Dominated by base case
  - All cases equal – we care about the depth