CSE 421
Algorithms

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Autumn 2015

Lecture 7

Announcements

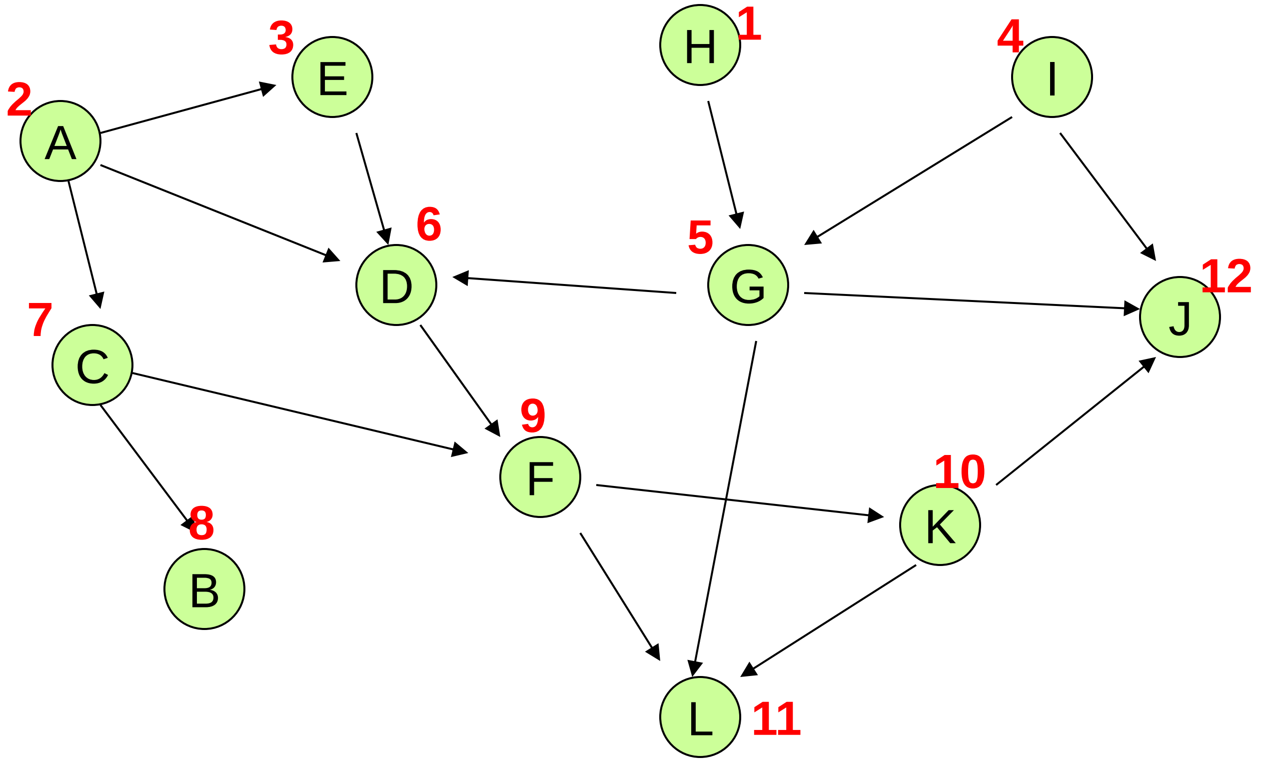
- Reading
  - For today, sections 4.1, 4.2, 4.4
  - For next week, sections 4.5, 4.7, 4.8
- Homework 3 is available
  - Random out-degree one graph
    - What does it look like
- Staff mailing list [Instructor + TAs]:
  - cse421-staff@cs.washington.edu

Highlight from last lecture: Topological Sort Algorithm

While there exists a vertex v with in-degree 0

Output vertex v

Delete the vertex v and all out going edges



Greedy Algorithms

Greedy Algorithms

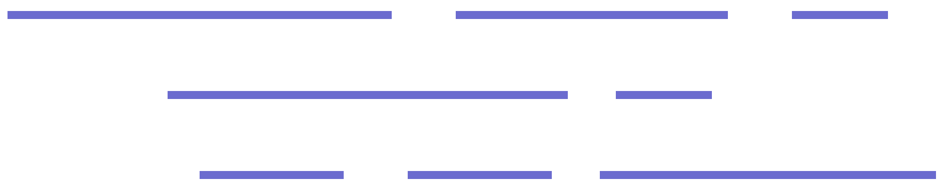
- Solve problems with the simplest possible algorithm
- The hard part: showing that something simple actually works
- Pseudo-definition
  - An algorithm is Greedy if it builds its solution by adding elements one at a time using a simple rule

Scheduling Theory

- Tasks
  - Processing requirements, release times, deadlines
- Processors
- Precedence constraints
- Objective function
  - Jobs scheduled, lateness, total execution time

Interval Scheduling

- Tasks occur at fixed times
- Single processor
- Maximize number of tasks completed



- Tasks {1, 2, . . . N}
- Start and finish times, s(i), f(i)

What is the largest solution?

Greedy Algorithm for Scheduling

Let T be the set of tasks, construct a set of independent tasks I, A is the rule determining the greedy algorithm

I = { }

While (T is not empty)

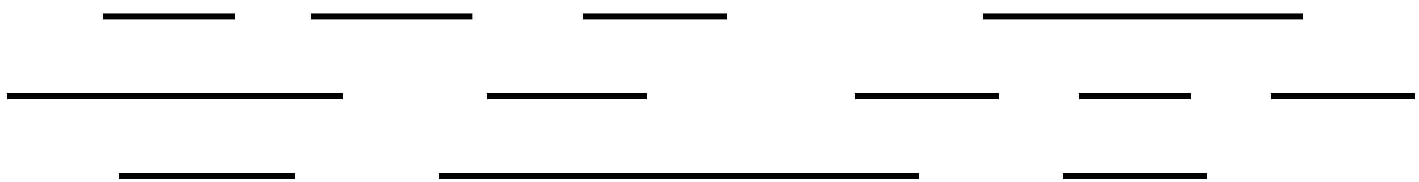
    Select a task t from T by a rule A

    Add t to I

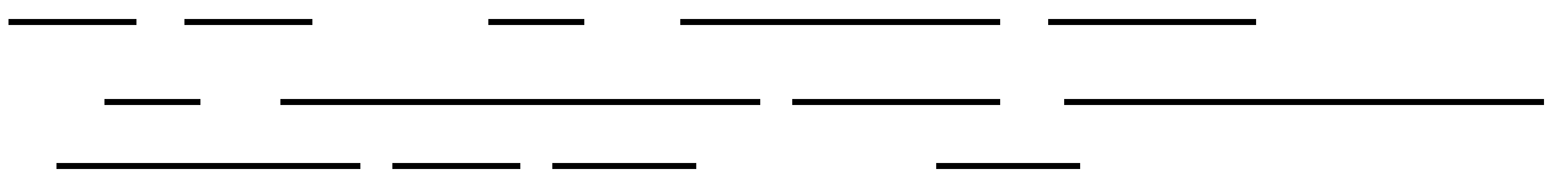
    Remove t and all tasks incompatible with t from T

Simulate the greedy algorithm for each of these heuristics

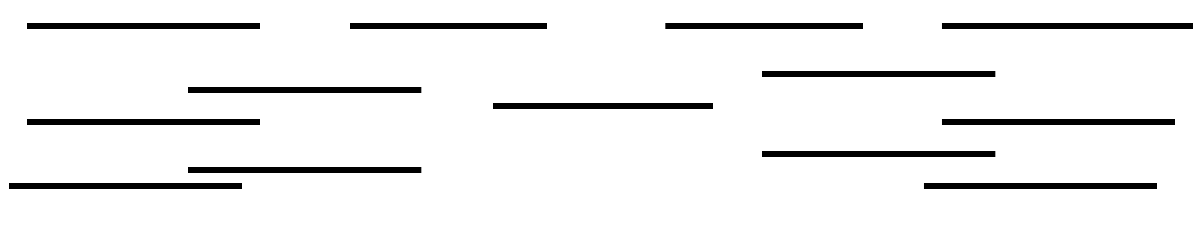
Schedule earliest starting task



Schedule shortest available task

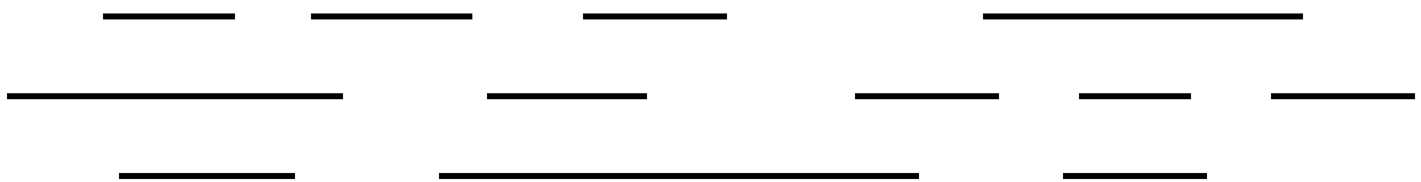


Schedule task with fewest conflicting tasks

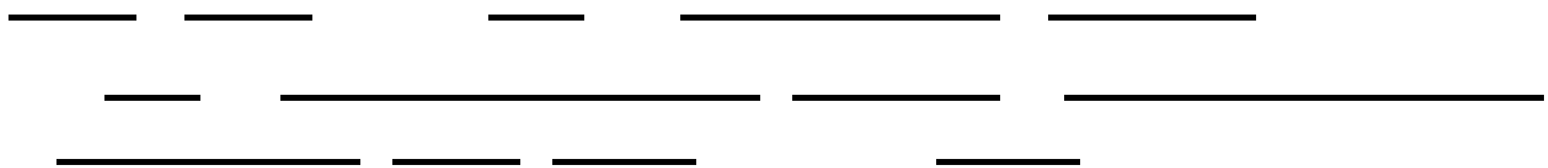


Greedy solution based on earliest finishing time

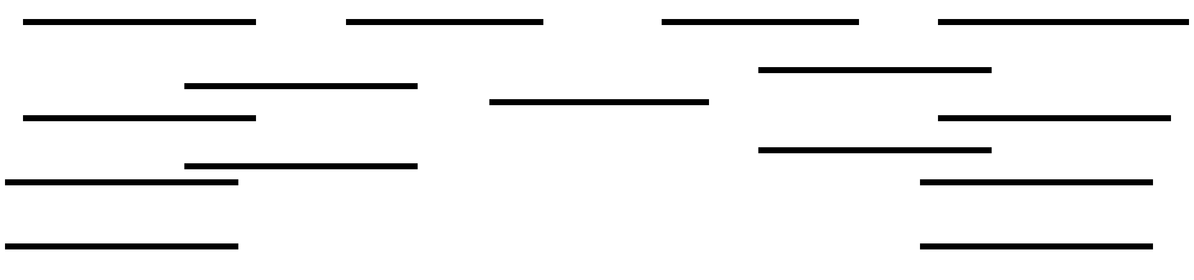
Example 1



Example 2



Example 3



Theorem: Earliest Finish Algorithm is Optimal

- Key idea: Earliest Finish Algorithm stays ahead
- Let $A = \{i_1, \ldots, i_k\}$ be the set of tasks found by EFA in increasing order of finish times
- Let $B = \{j_1, \ldots, j_m\}$ be the set of tasks found by a different algorithm in increasing order of finish times
- Show that for $r \le \min(k, m)$, $f(i_r) \le f(j_r)$

Stay ahead lemma

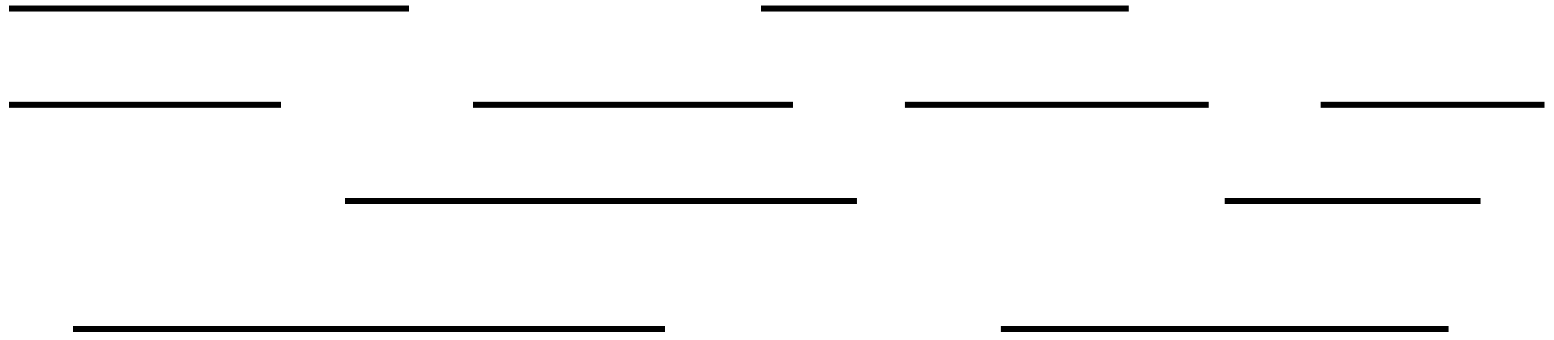
- A always stays ahead of B, $f(i_r) <= f(j_r)$
- Induction argument
  - $f(i_1) <= f(j_1)$
  - If $f(i_{r-1}) <= f(j_{r-1})$ then $f(i_r) <= f(j_r)$

Completing the proof

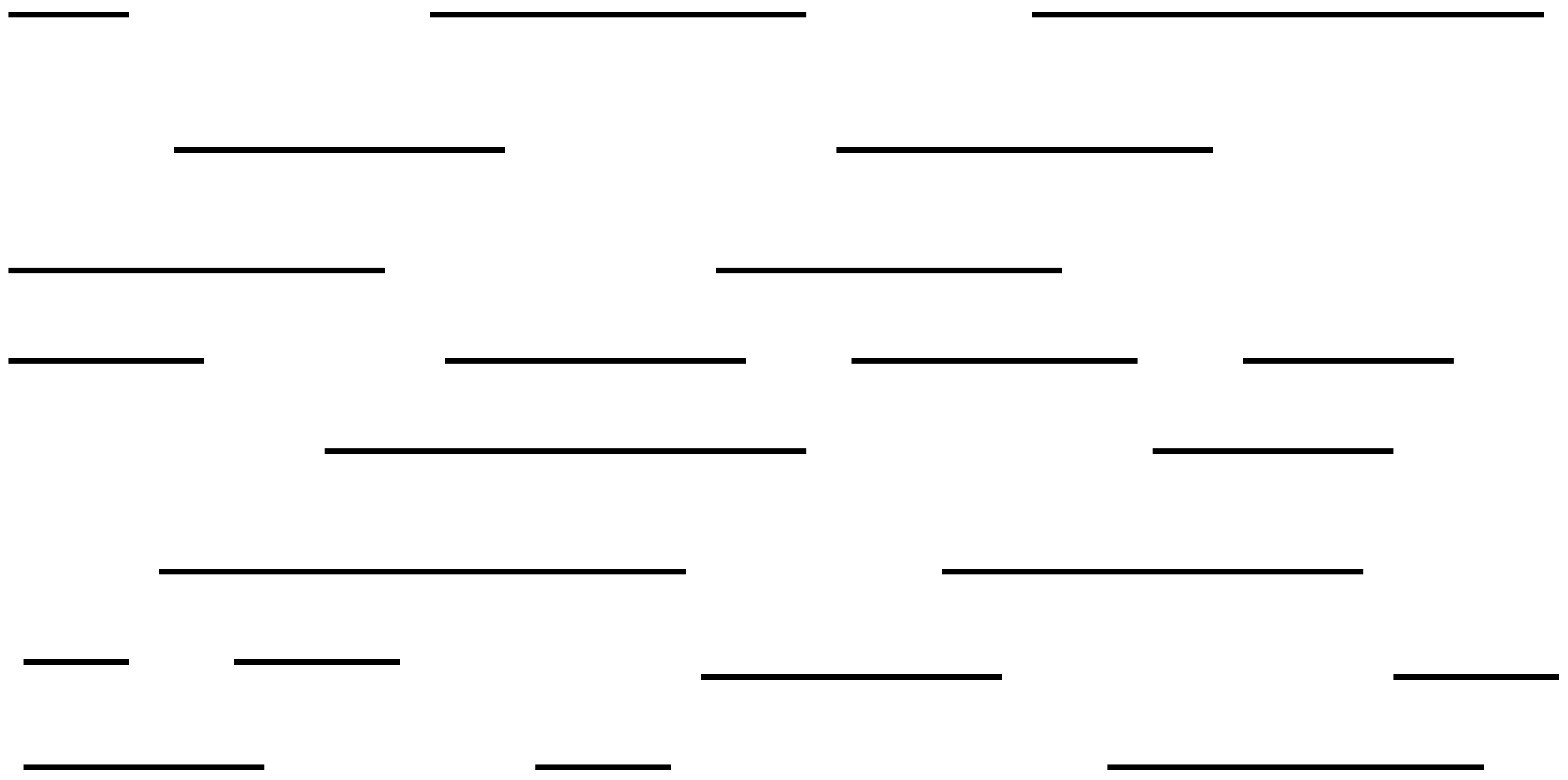
- Let A = $\{i_1, . . ., i_k\}$ be the set of tasks found by EFA in increasing order of finish times
- Let O = $\{j_1, . . ., j_m\}$ be the set of tasks found by an optimal algorithm in increasing order of finish times
- If $k < m$, then the Earliest Finish Algorithm stopped before it ran out of tasks

Scheduling all intervals

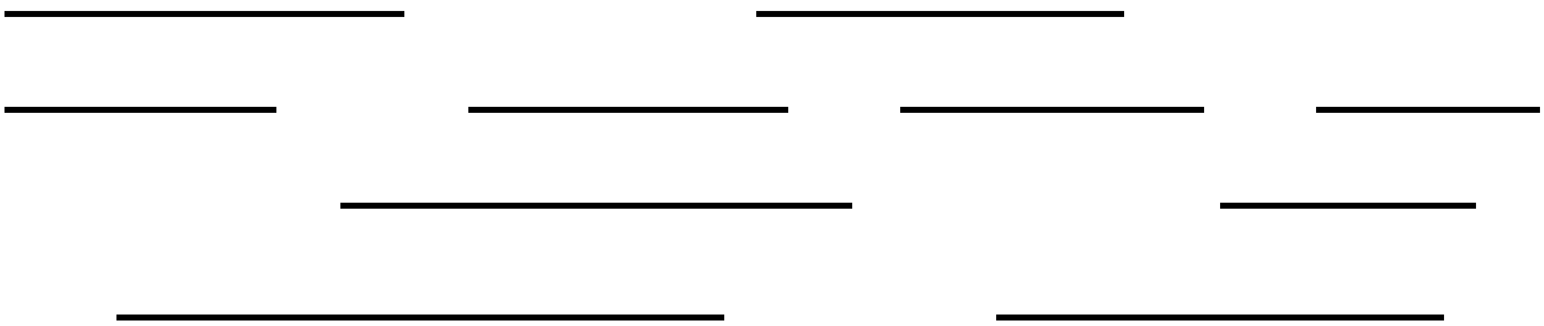
- Minimize number of processors to schedule all intervals



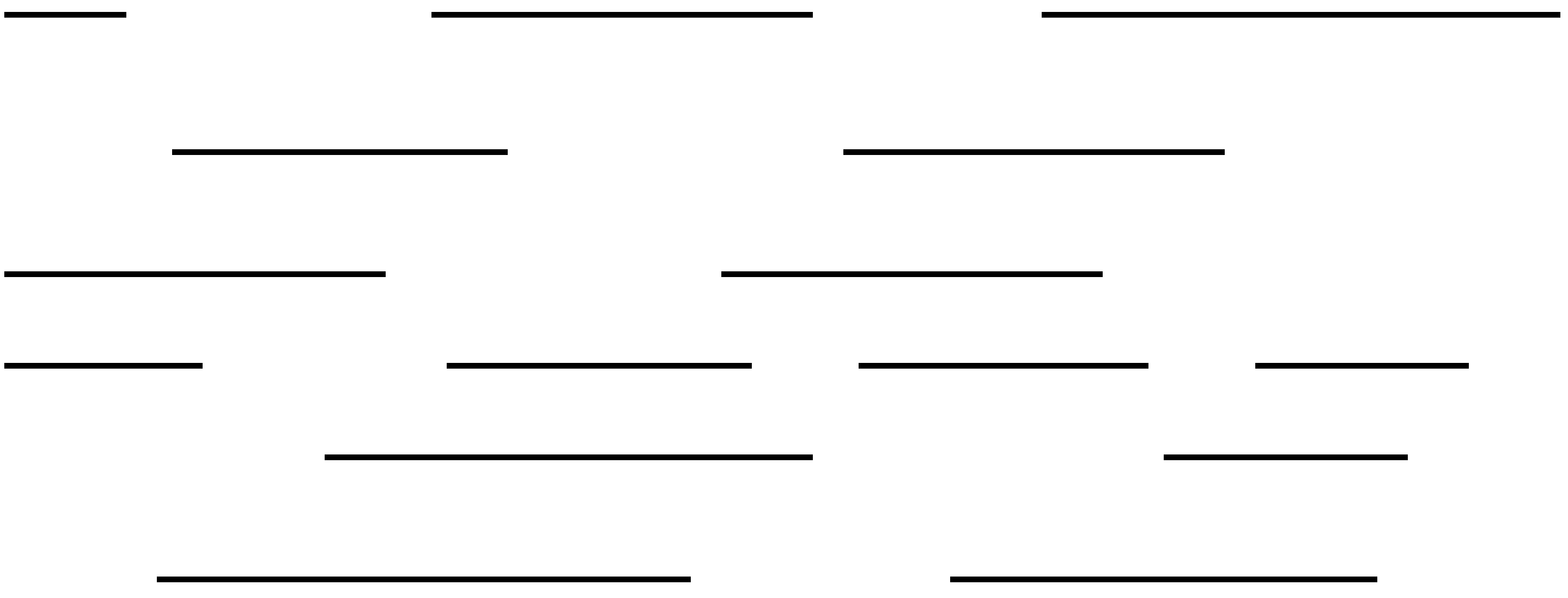
How many processors are needed for this example?



Prove that you cannot schedule this set of intervals with two processors



Depth: maximum number of intervals active



Algorithm

- Sort by start times
- Suppose maximum depth is d, create d slots
- Schedule items in increasing order, assign each item to an open slot

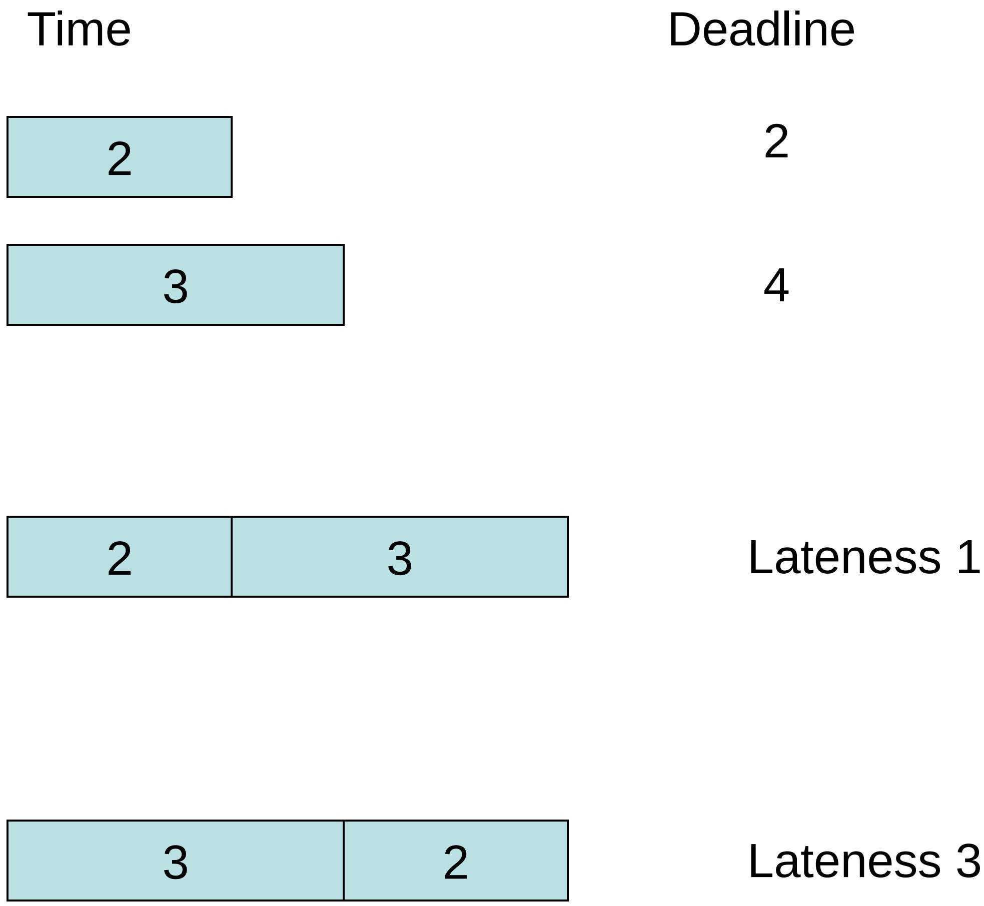
- Correctness proof: When we reach an item, we always have an open slot

Scheduling tasks

- Each task has a length t_i and a deadline d_i
- All tasks are available at the start
- One task may be worked on at a time
- All tasks must be completed

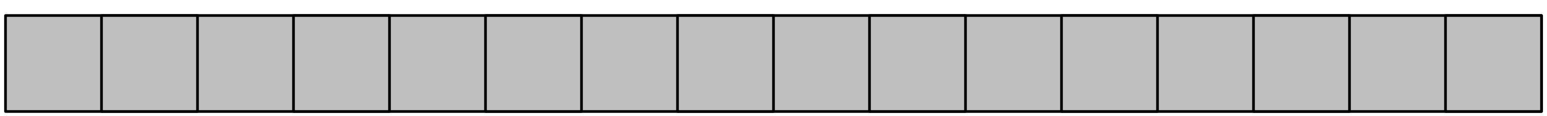
- Goal minimize maximum lateness
  - Lateness = $f_i - d_i$ if $f_i >= d_i$

Example



Determine the minimum lateness

Time	Deadline
2	6
3	4
4	5
5	12



Greedy Algorithm

- Earliest deadline first
- Order jobs by deadline

- This algorithm is optimal

Analysis

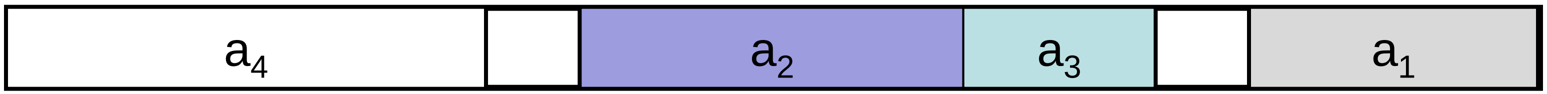
- Suppose the jobs are ordered by deadlines, $d_1 <= d_2 <= . . . <= d_n$
- A schedule has an *inversion* if job j is scheduled before i where j > i

- The schedule A computed by the greedy algorithm has no inversions.
- Let O be the optimal schedule, we want to show that A has the same maximum lateness as O

List the inversions

	Time	Deadline
a_1	3	4
a_2	4	5
a_3	2	6
a_4	5	12

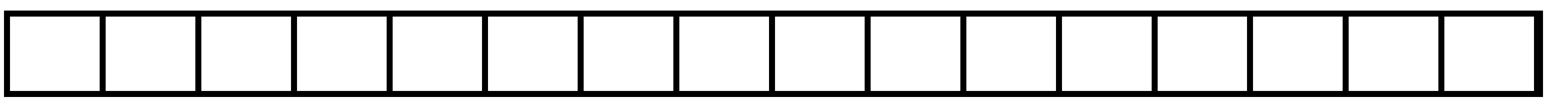
Lemma: There is an optimal schedule with no idle time



- It doesn't hurt to start your homework early!
- Note on proof techniques
  - This type of can be important for keeping proofs clean
  - It allows us to make a simplifying assumption for the remainder of the proof

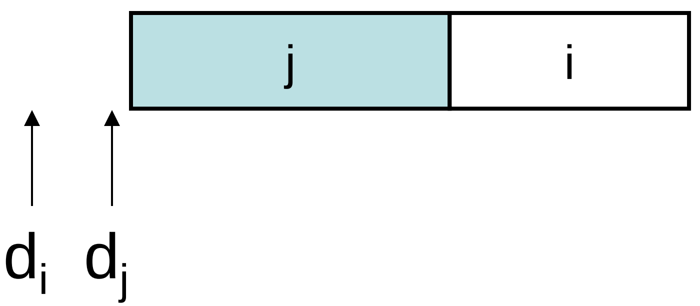
Lemma

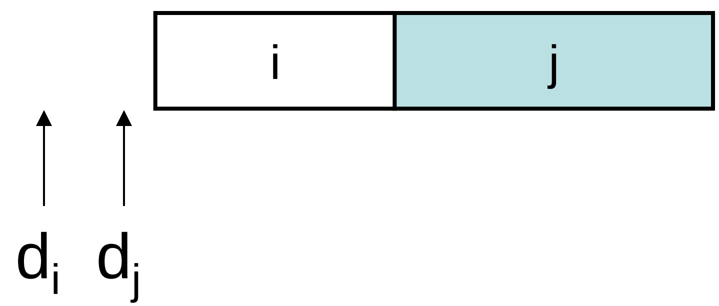
- If there is an inversion i, j, there is a pair of adjacent jobs i’, j’ which form an inversion



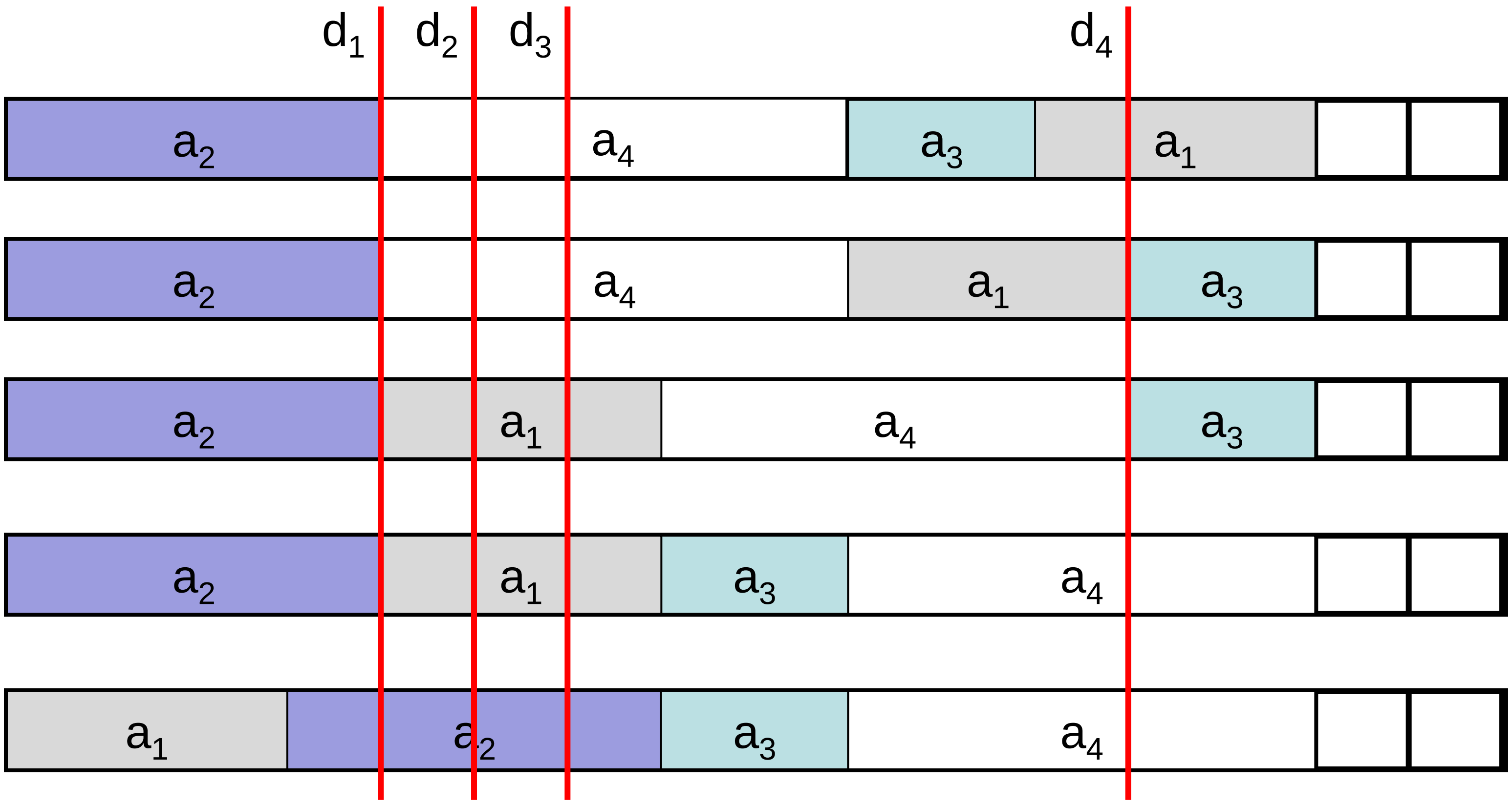
Interchange argument

- Suppose there is a pair of jobs i and j, with $d_i <= d_j$, and j scheduled immediately before i. Interchanging i and j does not increase the maximum lateness.





Proof by Bubble Sort



Determine maximum lateness

Real Proof

- There is an optimal schedule with no inversions and no idle time.
- Let O be an optimal schedule k inversions, we construct a new optimal schedule with k-1 inversions
- Repeat until we have an optimal schedule with 0 inversions
- This is the solution found by the earliest deadline first algorithm

Result

- Earliest Deadline First algorithm constructs a schedule that minimizes the maximum lateness