Homework 3, Due Wednesday October 21, 2015

Problem 1 (10 points):

Page 109, Exercise 8.

Problem 2 (10 points):

Page 110, Exercise 9.

Problem 3 (10 points):

Consider a directed graph on n vertices, where each vertex has exactly one out going edge. This graph consists of a collection of cycles as well as additional vertices that have paths to the cycles, which we will call the branches. We define the *weight* of the cycle to be the total number of vertices that are either on the cycle or on branches that are connected to the cycle.

Describe a linear time algorithm that identifies all of the cycles and computes the length and weight of each cycle.

Programming Problem4 (10 points) :

(10 points) Implement your algorithm for finding the cycles in an out-degree one graph. Your algorithm should be designed to work on very large graphs, e.g., with $n = 100,000,000$.

Write an input generator which creates completely random out-degree one graphs where each vertex points to another vertex chosen uniformly at random.

As the size of the problem increases - how does the number of cycles, and the length, and weight of the cycles change, when the input is a random graph with out-degree one?

You are free to write in any programming language you like. The quality of your algorithm may be graded, but the actual quality of the code will not be graded. The expectation is that you will write the algorithmic code yourself - but you can use other code or libraries for supporting operations.

Problem 5 (10 points):

Page 189, Exercise 3.

Problem 6 (10 points):

Page 190, Exercise 5. Justify that your algorithm is correct.