CSE 421: Algorithms

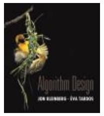
Winter 2014
Lecture 1: Introductions

course information

Instructor: James R. Lee (me!)

Teaching assistants: Armando J. Diaz Tolentino
Yanling He

Book: Algorithm Design



Homework: Due WED at start of class

Exams: Midterm and Final (TBD soon)

All course information at http://www.cs.washington.edu/421

a facebook post

Ian Simon
December 28, 2013

Math/algorithm question (to which I do not know the answer): I have a bunch of points on the unit circle. Can I find the median point in linear time? I can't even figure out a way to do this if the points are sorted, but I think it must be possible.

By median point I mean the point that minimizes the sum of distance along the circle to each of the other points. Assume I have an odd number of points, which I think means that the median must be one of my input points.

Follow-up question: I'm now going to add one point and remove one point. Can I update the median in constant time?

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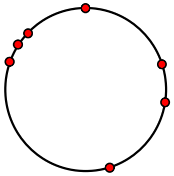
How can I find the median of points on a circle?

median on a circle

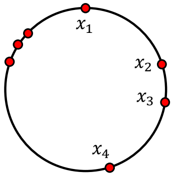


Input: Points $x_1, x_2, \ldots, x_n$ on the unit circle.

Let $sum(x) = d(x, x_1) + d(x, x_2) + \cdots + d(x, x_n)$

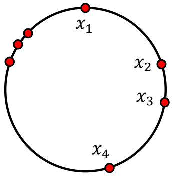
Find index j such that $sum(x_j)$ is minimal.

sorting the points



How long to sort the input points?

iterative computation?



a problem from Intel

Object recognition using labeled images.

matching residents to hospitals

Goal: Given a set of preferences among hospitals and medical school residents (graduating medical students), design a self-reinforcing admissions process.

matching residents to hospitals

Goal: Given a set of preferences among hospitals and medical school residents (graduating medical students), design a self-reinforcing admissions process.

Unstable pair: applicant x and hospital y are unstable if:
- x prefers y to their assigned hospital.
- y prefers x to one of its admitted residents.

Stable assignment. Assignment with no unstable pairs.
- Natural and desirable condition.
- Individual self-interest will prevent any applicant/hospital side deal from being made.

stable matching problem

Goal: Given n men and n women, find a "suitable" matching.

Participants rate members of opposite sex.
Each man lists women in order of preference: best to worst.
Each woman lists men in order of preference: best to worst.

	favorite ↓		least favorite ↓
	1st	2nd	3rd
Xavier	Amy	Brenda	Claire
Yuri	Brenda	Amy	Claire
Zoran	Amy	Brenda	Claire

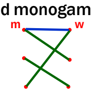
Men's Preference Profile

	favorite ↓		least favorite ↓
	1st	2nd	3rd
Amy	Yuri	Xavier	Zoran
Brenda	Xavier	Yuri	Zoran
Claire	Xavier	Yuri	Zoran

Women's Preference Profile

stable matching problem

- Perfect matching: everyone is matched monogamously.
  - Each man gets exactly one woman.
  - Each woman gets exactly one man.



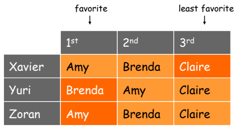
- Stability: no incentive for some pair of participants to undermine assignment by joint action.
  - In matching M, an unmatched pair m-w is unstable if man m and woman w prefer each other to current partners.
  - Unstable pair m-w could each improve by eloping.

- Stable matching: perfect matching with no unstable pairs.

- Stable matching problem: Given the preference lists of n men and n women, find a stable matching if one exists.

stable matching problem

Is assignment X-C, Y-B, Z-A stable?



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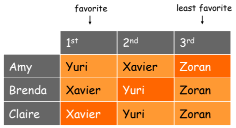
Men's Preference Profile



	favorite ↓		least favorite ↓
	1st	2nd	3rd
Amy	Yuri	Xavier	Zoran
Brenda	Xavier	Yuri	Zoran
Claire	Xavier	Yuri	Zoran

Women's Preference Profile

stable matching problem

Is assignment X-A, Y-B, Z-C stable?

favorite ↓ least favorite ↓

	1st	2nd	3rd
Xavier	Amy	Brenda	Claire
Yuri	Brenda	Amy	Claire
Zoran	Amy	Brenda	Claire

Men's Preference Profile

favorite ↓ least favorite ↓

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Women's Preference Profile

stable roommates

Q. Do stable matchings always exist?
A. Not obvious a priori.

Stable roommate problem.

- 2n people; each person ranks others from 1 to 2n-1.
- Assign roommate pairs so that no unstable pairs.

	1st	*2nd*	*3rd*
Adam	B	C	D
Bob	C	A	D
Chris	A	B	D
David	A	B	C

A-B, C-D ⇒ B-C unstable
A-C, B-D ⇒ A-B unstable
A-D, B-C ⇒ A-C unstable

propose-and-reject algorithm

Propose-and-reject algorithm. [Gale-Shapley 1962]
Intuitive method that is guaranteed to find a stable matching.

```
Initialize each person to be free
while (some man is free and hasn't proposed to every woman) {
    Choose such a man m
    w = 1st woman on m's list to whom m has not yet proposed
    if (w is free)
        assign m and w to be engaged
    else if (w prefers m to her fiancé m')
        assign m and w to be engaged, and m' to be free
    else
        w rejects m
}
```

http://mathsite.math.berkeley.edu/smp/smp.html
http://www.cs.columbia.edu/~evs/intro/stable/Stable.html
http://demonstrations.wolfram.com/StableMarriages/

proof of correctness: termination

- **Observation 1.** Men propose to women in decreasing order of preference.
- **Observation 2.** Once a woman is matched, she never becomes unmatched; she only "trades up."
- **Claim.** Algorithm terminates after at most n^2 iterations of while loop.
- Proof. Each time through the while loop a man proposes to a new woman. There are only n^2 possible proposals. ▪

	1st	2nd	3rd	4th	5th
Victor	A	B	C	D	E
Walter	B	C	D	A	E
Xavier	C	D	A	B	E
Yuri	D	A	B	C	E
Zoran	A	B	C	D	E

	1st	2nd	3rd	4th	5th
Amy	W	X	Y	Z	V
Brenda	X	Y	Z	V	W
Claire	Y	Z	V	W	X
Diane	Z	V	W	X	Y
Erika	V	W	X	Y	Z

$n(n-1) + 1$ proposals required

proof of correctness: perfection

- **Claim:** All men and women get matched.
- Proof. (by contradiction)
  - Suppose, for sake of contradiction, that Zoran is not matched upon termination of algorithm.
  - Then some woman, say Amy, is not matched upon termination.
  - By Observation 2 (only trading up, never becoming unmatched), Amy was never proposed to.
  - But, Zoran proposes to everyone, since he ends up unmatched. ▪

proof of correctness: stability

Claim: No unstable pairs.
Proof. (by contradiction)

- Suppose A-Z is an unstable pair: each prefers each other to partner in Gale-Shapley matching S*.

summary

- Stable matching problem. Given n men and n women, and their preferences, find a stable matching if one exists.
- Gale-Shapley algorithm. Guarantees to find a stable matching for any problem instance.
- How to implement GS algorithm efficiently?
- If there are multiple stable matchings, which one does GS find?