CSE 421: Review

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Complexity, I

Asymptotic Analysis

Best/average/**worst** cases

Upper/Lower Bounds

Big O, Theta, Omega

- definitions; intuition

Analysis methods

- loops
- recurrence relations
- common data structures, subroutines

Graph Algorithms

Graphs

- Representation (edge list/adjacency matrix)
- Breadth/depth first search
- Connected components
- Shortest paths/bipartitness/2-Colorability
- DAGS and topological ordering
- DFS/articulation points/biconnected components

Examples

Greedy

- Interval Scheduling Problems (3)
- Huffman Codes
- Examples where greedy fails (stamps/change, scheduling, knap, RNA,…)

Examples

Divide & Conquer

- Merge sort
- Counting Inversions
- Closest pair of points
- Integer multiplication (Karatsuba)
- Matrix multiplication (Strassen)
- Powering

Design Paradigms

Greedy

- emphasis on correctness arguments, e.g. stay ahead, structural characterizations, exchange arguments

Divide & Conquer

- recursive solution, superlinear work, balanced subproblems, recurrence relations, solutions, Master Theorem

Dynamic Programming

- recursive solution, redundant subproblems, few
- do all in careful order and tabulate; OPT table
- (usually far superior to "memoization")

Examples

Dynamic programming

- Fibonacci
- Making change/Stamps, Knapsack
- Weighted Interval Scheduling
- RNA
- String Alignment

OPT function

Examples & Concepts

Flow and matching

Residual graph, augmenting paths, max-flow/min-cut, Ford-Fulkerson and Edmonds-Karp algorithms, (preflow-push), integrality, reductions to flow, e.g. bipartite matching

Complexity, II

P vs NP

- Big-O and poly vs exponential growth
- Definition of NP – hints/certificates and verifiers
- Example problems from slides, reading & hw
  - SAT, 3-SAT, circuit SAT, vertex cover, quadratic Diophantine equations, clique, independent set, TSP, Hamilton cycle, coloring, max cut, knapsack
- P $\subseteq$ NP $\subseteq$ Exp (and worse)
- Definition(s) of (polynomial time) reduction
- SAT $\leq_p$ e.g., IndpSet, Knap, Ham, 3color: how, correctness, $\leq_p$, implications
- Definition of NP-completeness
- NP-completeness proofs
- 2x, 1.5x approximations to Euclidean TSP

And see how relevant
it is to your daily life!

Classic Nintendo Games are (NP-)Hard

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Abstract

We prove NP-hardness results for five of Nintendo's largest video game franchises: Mario, Donkey Kong, Legend of Zelda, Metroid, and Pokémon. Our results apply to Super Mario Bros. 1, 3, Lost Levels, and Super Mario World; Donkey Kong Country 1–3; all Legend of Zelda games except Zelda II: The Adventure of Link; all Metroid games; and all Pokémon role-playing games. For Mario and Donkey Kong, we show NP-completeness. In addition, we observe that several games in the Zelda series are PSPACE-complete.

Final Exam Mechanics

Closed book, 1 pg notes (8.5x11, 2 sides, handwritten)

(no bluebook needed; scratch paper may be handy; calculators unnecessary)

Comprehensive: All topics covered

- assigned reading
- slides
- homework & solutions

Some Typical Exam Questions

Give O( ) bound on 17n*(n-3+logn), or on code `{for i=1 …}}`

True/False: If X is $O(n^2)$, then it's rarely more than n^3 +14 steps.

Explain why a given greedy alg is/isn't correct

Give a run time recurrence for a recursive alg, or solve a simple one

Simulate any of the algs we've studied

Give an alg for problem X, maybe a variant of one we've studied, or prove it's in NP

Understand parts of correctness proof for an algorithm or reduction

Implications of NP-completeness

Reductions

NP-completeness proofs

~~Hell's library~~ → 421 Final

Good Luck!