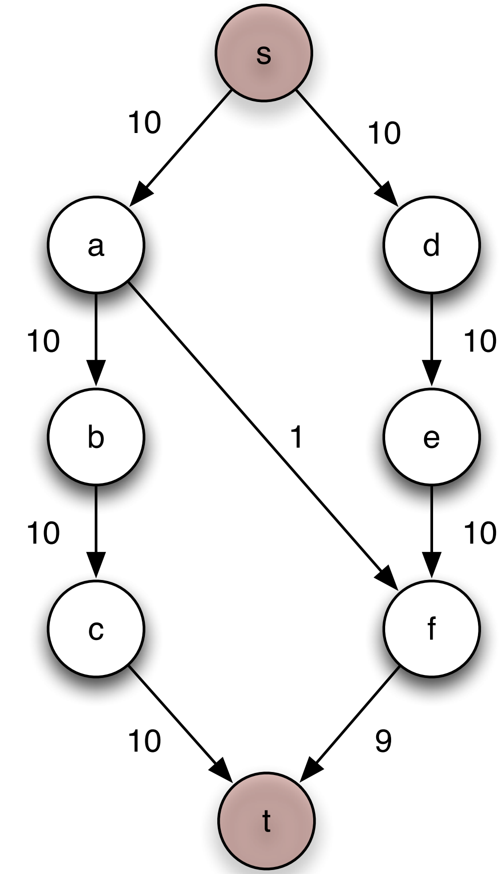
Illustrating the Edmonds-Karp-Dinitz Max Flow Algorithm.

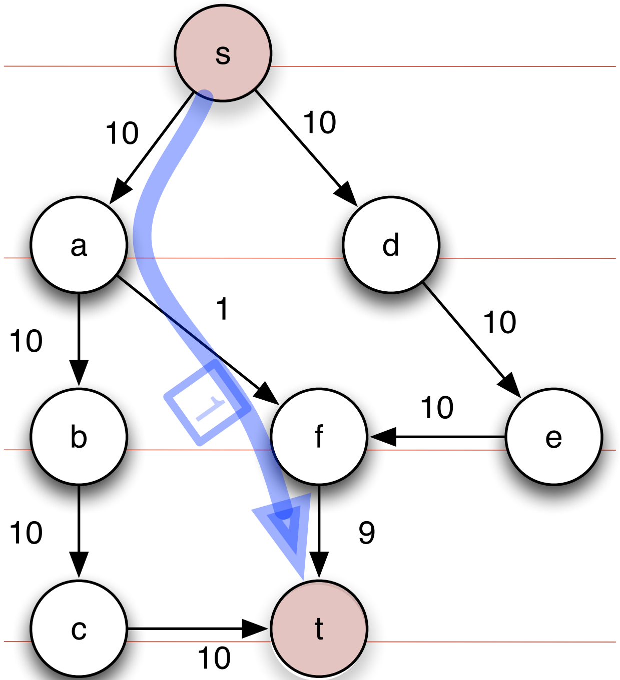
Figures show successive stages of the E-K-D algorithm, including the 4 augmenting paths selected, while solving a particular max-flow problem. "Real" edges in the graph are shown in black, and dashed if their residual capacity is zero. Green residual edges are the back edges created to allow "undo" of flow on a "real" edge. Each graph containing an augmenting path is drawn twice – first as a "plain" graph, then showing the layering induced by breadth-first search, together with an augmenting path chosen at that stage (light blue). G4 has no remaining augmenting paths (edges from s are saturated); G5 is the resulting max flow, with each edge annotated by "flow" / "capacity".

Note how successive augmentations push nodes steadily farther from s, and especially that (undirected) edge {a,f} is the "critical" edge twice – first in G0, when a is at depth 1 in the BFS tree, and again in G3 when f (not a) is at depth 3, which allows us to undo the "mistake" of sending any flow through this edge.

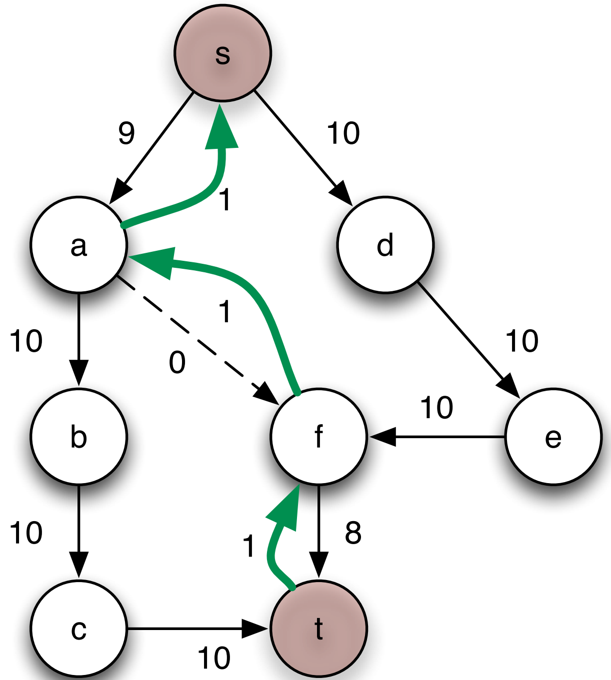
Edge capacities > 1 could be increased by any value C greater than 1 without fundamentally altering the series of graphs shown. Hence, Ford-Fulkerson (lacking the E-K-D shortest path innovation) might use ≈ C augmentations on G0, instead of 4.



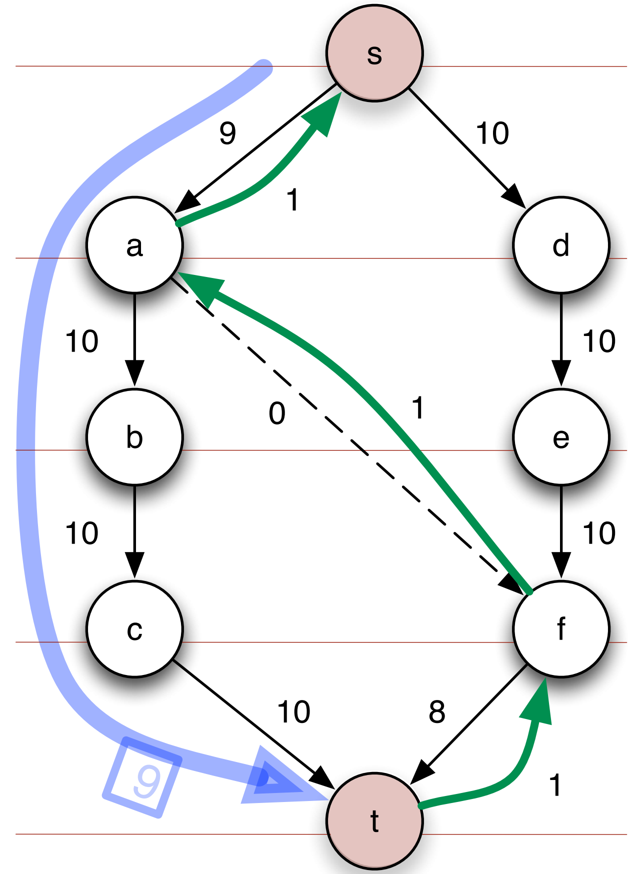
G_0: the flow problem



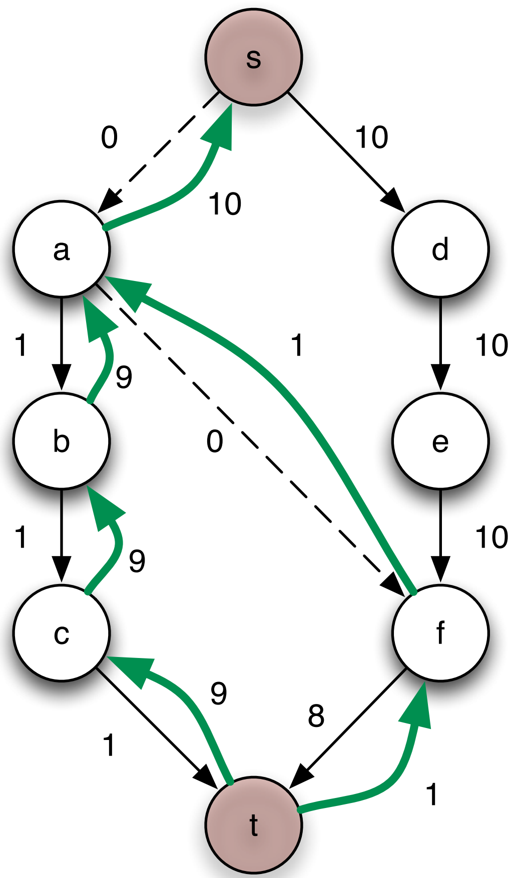
G_0: BFS layering + Aug Path



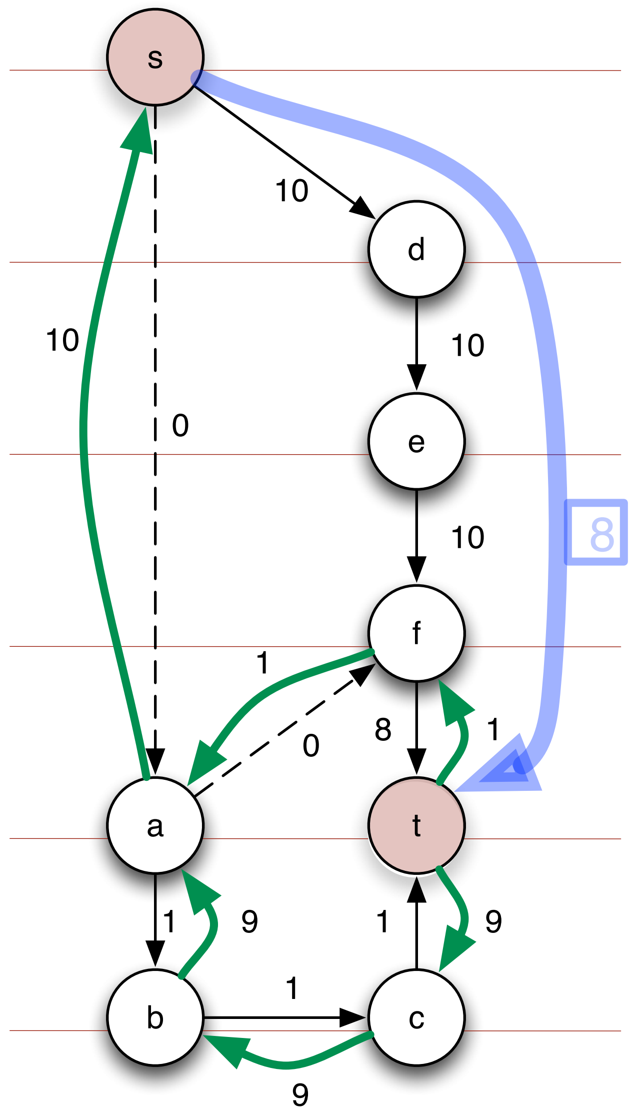
G_1: 1st Residual Graph



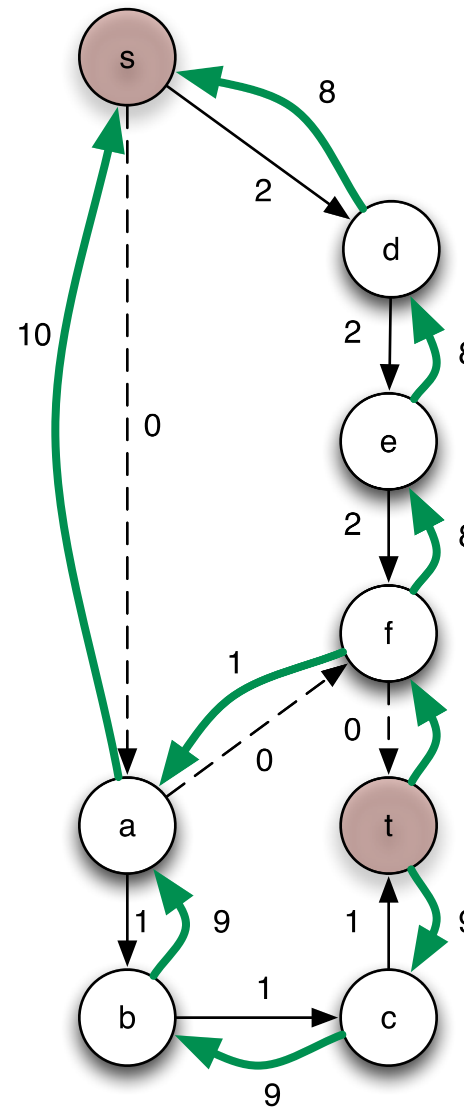
G_1: BFS layering + Aug Path



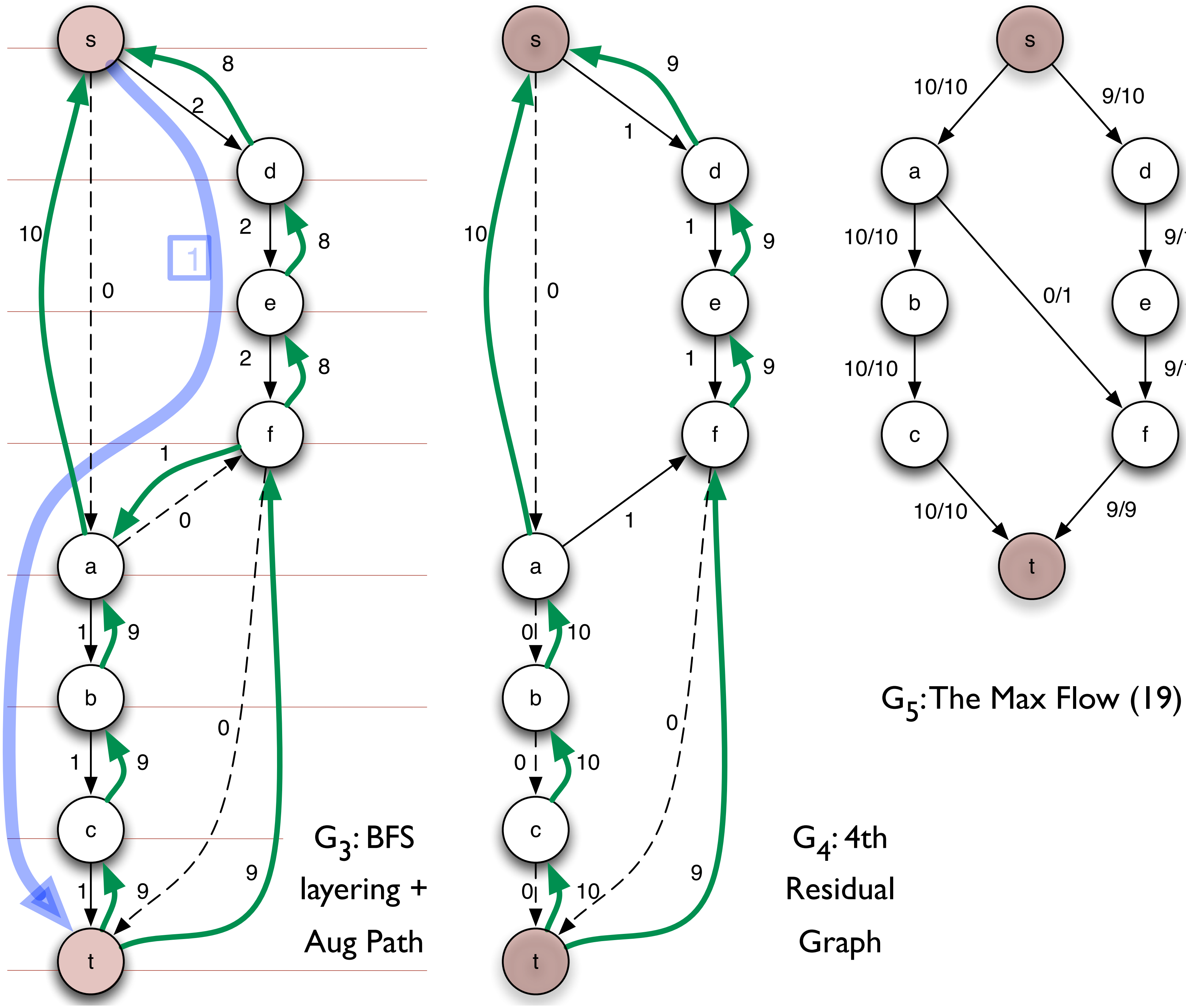
G_2: 2nd Residual Graph



G_2: BFS layering + Aug Path



G_3: 3rd Residual Graph



G_3: BFS layering + Aug Path

G_4: 4th Residual Graph

G_5:The Max Flow (19)