CSE 421
Algorithms

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Lecture 23

Network Flow Applications

Today's topics

- Ford Fulkerson Performance
- Problem Reductions
  - – Undirected Flow to Flow
  - – Bipartite Matching
  - – Disjoint Path Problem
- Baseball Scheduling

- Reading: 7.5, 7.6, 7.12

Ford-Fulkerson Algorithm

while not done

Construct residual graph G_R

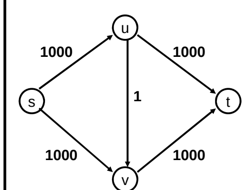
Find an s-t path P in G_R with capacity b > 0

Add b units along in G

If the sum of the capacities of edges leaving S is at most C, then the algorithm takes at most C iterations

Performance

- The worst case performance of the Ford-Fulkerson algorithm is horrible



Better methods of finding augmenting paths

- Find the maximum capacity augmenting path
  - – $O(m^2 \log(C))$ time algorithm for network flow
- Find the shortest augmenting path
  - – $O(m^2 n)$ time algorithm for network flow
- Find a blocking flow in the residual graph
  - – O(mnlog n) time algorithm for network flow

Problem Reduction

- Reduce Problem A to Problem B
  - – Convert an instance of Problem A to an instance Problem B
  - – Use a solution of Problem B to get a solution to Problem A
- Practical
  - – Use a program for Problem B to solve Problem A
- Theoretical
  - – Show that Problem B is at least as hard as Problem A

Problem Reduction Examples

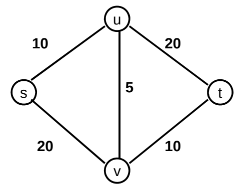
- Reduce the problem of finding the Maximum of a set of integers to finding the Minimum of a set of integers

Find the maximum of: 8, -3, 2, 12, 1, -6

Construct an equivalent minimization problem

Undirected Network Flow

- Undirected graph with edge capacities
- Flow may go either direction along the edges (subject to the capacity constraints)



Construct an equivalent flow problem

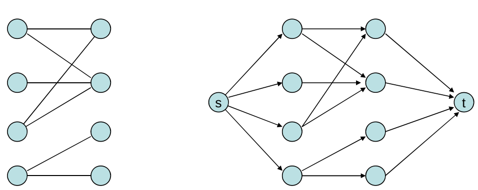
Bipartite Matching

- A graph G=(V,E) is bipartite if the vertices can be partitioned into disjoints sets X,Y
- A matching M is a subset of the edges that does not share any vertices
- Find a matching as large as possible

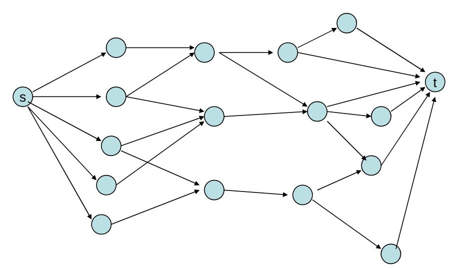
Application

- A collection of teachers
- A collection of courses
- And a graph showing which teachers can teach which courses

Converting Matching to Network Flow



Finding edge disjoint paths



Construct a maximum cardinality set of edge disjoint paths

Theorem

- The maximum number of edge disjoint paths equals the minimum number of edges whose removal separates s from t

Baseball elimination

- Can the Dinosaurs win the league?
- Remaining games:
  - AB, AC, AD, AD, AD, BC, BC, BC, BD, CD

	W	L
Ants	4	2
Bees	4	2
Cockroaches	3	3
Dinosaurs	1	5

A team wins the league if it has strictly more wins than any other team at the end of the season
A team ties for first place if no team has more wins, and there is some other team with the same number of wins

Baseball elimination

- Can the Fruit Flies win the league?
- Remaining games:
  - AC, AD, AD, AD, AF, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, BF, CE, CE, CE, CF, CF, DE, DF, EF, EF

	W	L
Ants	17	12
Bees	16	7
Cockroaches	16	7
Dinosaurs	14	13
Earthworms	14	10
Fruit Flies	12	15

Assume Fruit Flies win remaining games

- Fruit Flies are tied for first place if no team wins more than 19 games
- Allowable wins
  - Ants (2)
  - Bees (3)
  - Cockroaches (3)
  - Dinosaurs (5)
  - Earthworms (5)
- 18 games to play
  - AC, AD, AD, AD, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, CE, CE, CE, DE

	W	L
Ants	17	13
Bees	16	8
Cockroaches	16	9
Dinosaurs	14	14
Earthworms	14	12
Fruit Flies	19	15

Remaining games

AC, AD, AD, AD, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, CE, CE, CE, DE