CSE 421
Algorithms

Richard Anderson

Winter 2009

Lecture 1

CSE 421 Course Introduction

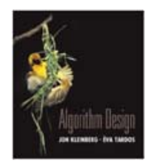
- CSE 421, Introduction to Algorithms
  - MWF, 1:30-2:20 pm
  - EEB 037
- Instructor
  - Richard Anderson, anderson@cs.washington.edu
  - Office hours:
    - CSE 582
    - Monday, 3:00-3:50 pm, Thursday, 11:00-11:50 am
- Teaching Assistant
  - Aeron Bryce, paradoxa@cs.washington.edu
  - Office hours:
    - CSE 216
    - Monday, 12:30-1:20 pm, Tuesday, 12:30-1:20 pm

Announcements

- It's on the web.
- Homework due Wednesdays
  - HW 1, Due January 14, 2009
  - It's on the web (or will be soon)
- Subscribe to the mailing list

Text book

- Algorithm Design
- Jon Kleinberg, Eva Tardos
- Read Chapters 1 & 2
- Expected coverage:
  - Chapter 1 through 7



Course Mechanics

- Homework
  - Due Wednesdays
  - About 5 problems + E.C.
  - Target: 1 week turnaround on grading
- Exams (In class)
  - Midterm, Monday, Feb 9 (probably)
  - Final, Monday, March 16, 2:30-4:20 pm
- Approximate grade weighting
  - HW: 50, MT: 15, Final: 35
- Course web
  - Slides, Handouts, Recorded Lectures from 2006

All of Computer Science is the Study of Algorithms

How to study algorithms

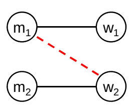
- Zoology
- Mine is faster than yours is
- Algorithmic ideas
  - Where algorithms apply
  - What makes an algorithm work
  - Algorithmic thinking

Introductory Problem: Stable Matching

- Setting:
  - Assign TAs to Instructors
  - Avoid having TAs and Instructors wanting changes
    - E.g., Prof A. would rather have student X than her current TA, and student X would rather work for Prof A. than his current instructor.

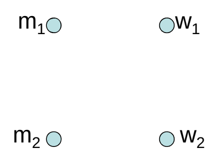
Formal notions

- Perfect matching
- Ranked preference lists
- Stability



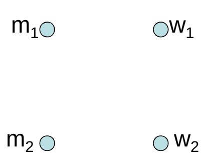
Example (1 of 3)

m_1: w_1 w_2
m_2: w_2 w_1
w_1: m_1 m_2
w_2: m_2 m_1



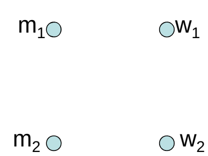
Example (2 of 3)

m_1: w_1 w_2
m_2: w_1 w_2
w_1: m_1 m_2
w_2: m_1 m_2



Example (3 of 3)

m_1: w_1 w_2
m_2: w_2 w_1
w_1: m_2 m_1
w_2: m_1 m_2



Formal Problem

- Input
  - Preference lists for $m_1, m_2, \ldots, m_n$
  - Preference lists for $w_1, w_2, \ldots, w_n$
- Output
  - Perfect matching M satisfying stability property:

> If $(m', w') \in M$ and $(m'', w'') \in M$ then
> (m' prefers w' to w'') or (w'' prefers m'' to m')

Idea for an Algorithm

m proposes to w

- If w is unmatched, w accepts
- If w is matched to m_2
  - If w prefers m to m_2 w accepts m, dumping m_2
  - If w prefers m_2 to m, w rejects m

Unmatched m proposes to the highest w on its preference list that it has not already proposed to

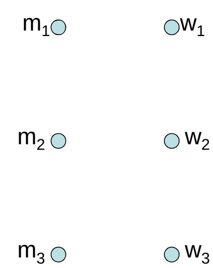
Algorithm

Initially all m in M and w in W are free
While there is a free m
    w highest on m's list that m has not proposed to
    if w is free, then match (m, w)
    else
        suppose (m_2, w) is matched
        if w prefers m to m_2
            unmatch (m_2, w)
            match (m, w)

Example

m_1: w_1 w_2 w_3
m_2: w_1 w_3 w_2
m_3: w_1 w_2 w_3

w_1: m_2 m_3 m_1
w_2: m_3 m_1 m_2
w_3: m_3 m_1 m_2



Does this work?

- Does it terminate?
- Is the result a stable matching?
- Begin by identifying invariants and measures of progress
  - m's proposals get worse (have higher m-rank)
  - Once w is matched, w stays matched
  - w's partners get better (have lower w-rank)

Claim: The algorithm stops in at most n^2 steps

When the algorithms halts, every w is matched

Why?

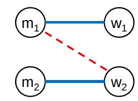
Hence, the algorithm finds a perfect matching

The resulting matching is stable

Suppose

$(m_1, w_1) \in M$, $(m_2, w_2) \in M$

m_1 prefers w_2 to w_1



How could this happen?

Result

- Simple, $O(n^2)$ algorithm to compute a stable matching
- Corollary
  - A stable matching always exists

A closer look

Stable matchings are not necessarily fair

m_1: w_1 w_2 w_3

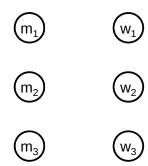
m_2: w_2 w_3 w_1

m_3: w_3 w_1 w_2

w_1: m_2 m_3 m_1

w_2: m_3 m_1 m_2

w_3: m_1 m_2 m_3



How many stable matchings can you find?

Algorithm under specified

- Many different ways of picking m's to propose
- Surprising result
  - All orderings of picking free m's give the same result
- Proving this type of result
  - Reordering argument
  - Prove algorithm is computing something mores specific
    - Show property of the solution – so it computes a specific stable matching

Proposal Algorithm finds the best possible solution for M

Formalize the notion of best possible solution:

(m, w) is valid if (m, w) is in some stable matching

best(m): the highest ranked w for m such that (m, w) is valid

S* = {(m, best(m))}

Every execution of the proposal algorithm computes S*

Proof

See the text book – pages 9 – 12

Related result: Proposal algorithm is the worst case for W

Algorithm is the M-optimal algorithm

Proposal algorithms where w's propose is W-Optimal

Best choices for one side may be bad for the other

Design a configuration for problem of size 4:

- M proposal algorithm:
  - All m's get first choice, all w's get last choice
- W proposal algorithm:
  - All w's get first choice, all m's get last choice

m_1:

m_2:

m_3:

m_4:

w_1:

w_2:

w_3:

w_4:

But there is a stable second choice

Design a configuration for problem of size 4:

- M proposal algorithm:
  - All m's get first choice, all w's get last choice
- W proposal algorithm:
  - All w's get first choice, all m's get last choice
- There is a stable matching where everyone gets their second choice

m_1:

m_2:

m_3:

m_4:

w_1:

w_2:

w_3:

w_4:

Key ideas

- Formalizing real world problem
  - Model: graph and preference lists
  - Mechanism: stability condition
- Specification of algorithm with a natural operation
  - Proposal
- Establishing termination of process through invariants and progress measure
- Under specification of algorithm
- Establishing uniqueness of solution