CSE 421
Algorithms

Richard Anderson
Autumn 2006
Lecture 1

Course Introduction

- Instructor
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  - Office hours:
    - CSE 582
    - Tuesday, 2:30-3:20 pm, Friday, 2:30-3:20 pm.
- Teaching Assistant
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  - Office hours:
    - Monday, 12:30-1:20 pm, Tuesday, 4:30-5:20 pm

Announcements

- It's on the web.
- Homework 1, Due October 4
  - It's on the web
- Subscribe to the mailing list

Text book

- Algorithm Design
- Jon Kleinberg, Eva Tardos

- Read Chapters 1 & 2

CSE 421 at Beihang University

- Parallel Course offering at Beihang University in Beijing
- Course offered with Tutored Video Instruction
  - Lectures recorded at University of Washington
  - Lectures shown in the classroom with facilitators
    - Student initiated discussion
    - Instructor initiated discussion
    - Classroom Activities

Beihang University Students

Classroom Interaction

- Classroom Presenter
- Tablet PC's to support active learning
- Educational basis
  - Classroom Feedback
  - Active Learning
  - Pedagogical Goals
- Tablets will be used in UW class about once a week

All of Computer Science is the Study of Algorithms

How to study algorithms

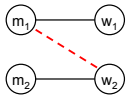
- Zoology
- Mine is faster than yours is
- Algorithmic ideas
  - Where algorithms apply
  - What makes an algorithm work
  - Algorithmic thinking

Introductory Problem: Stable Matching

- Setting:
  - Assign TAs to Instructors
  - Avoid having TAs and Instructors wanting changes
    - E.g., Prof A. would rather have student X than her current TA, and student X would rather work for Prof A. than his current instructor.

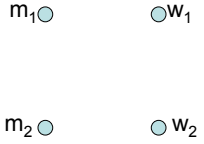
Formal notions

- Perfect matching
- Ranked preference lists
- Stability



Example (1 of 3)

- m_1: w_1 w_2
- m_2: w_2 w_1
- w_1: m_1 m_2
- w_2: m_2 m_1



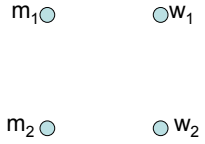
Example (2 of 3)

- m_1: w_1 w_2
- m_2: w_1 w_2
- w_1: m_1 m_2
- w_2: m_1 m_2

Find a stable matching

Example (3 of 3)

- m_1: w_1 w_2
- m_2: w_2 w_1
- w_1: m_2 m_1
- w_2: m_1 m_2



Intuitive Idea for an Algorithm

- m proposes to w
  - If w is unmatched, w accepts
  - If w is matched to m_2
    - If w prefers m to m_2, w accepts
    - If w prefers m_2 to m, w rejects
- Unmatched m proposes to highest w on its preference list that m has not already proposed to

Algorithm

Initially all m in M and w in W are free
While there is a free m
    w highest on m's list that m has not proposed to
    if w is free, then match (m, w)
    else
        suppose (m_2, w) is matched
        if w prefers m to m_2
            unmatch (m_2, w)
            match (m, w)

Example

m_1: w_1 w_2 w_3
m_2: w_1 w_3 w_2
m_3: w_1 w_2 w_3

w_1: m_2 m_3 m_1
w_2: m_3 m_1 m_2
w_3: m_3 m_1 m_2

Does this work?

- Does it terminate?
- Is the result a stable matching?
- Begin by identifying invariants and measures of progress
  - m's proposals get worse
  - Once w is matched, w stays matched
  - w's partners get better (have lower w-rank)

Claim: The algorithm stops in at most n^2 steps

- Why?

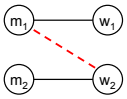
When the algorithms halts, every w is matched

- Why?

- Hence, the algorithm finds a perfect matching

The resulting matching is stable

- Suppose
  - m_1 prefers w_2 to w_1
- How could this happen?



Result

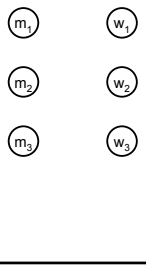
- Simple, $O(n^2)$ algorithm to compute a stable matching
- Corollary
  - A stable matching always exists

A closer look

- Stable matchings are not necessarily fair

m_1: w_1 w_2 w_3
m_2: w_2 w_3 w_1
m_3: w_3 w_1 w_2

w_1: m_2 m_3 m_1
w_2: m_3 m_1 m_2
w_3: m_1 m_2 m_3



How many stable matchings can you find?

Algorithm under specified

- Many different ways of picking m's to propose
- Surprising result
  - All orderings of picking free m's give the same result
- Proving this type of result
  - Reordering argument
  - Prove algorithm is computing something mores specific
    - Show property of the solution – so it computes a specific stable matching

Proposal Algorithm finds the best possible solution for M

- Formalize the notion of best possible solution
- (m, w) is valid if (m, w) is in some stable matching
- best(m): the highest ranked w for m such that (m, w) is valid
- S* = {(m, best(m)}
- Every execution of the proposal algorithm computes S*

Proof

- See the text book – pages 9 – 12
- Related result: Proposal algorithm is the worst case for W
- Algorithm is the M-optimal algorithm
- Proposal algorithms where w's propose is W-Optimal

Best choices for one side are bad for the other

- Design a configuration for problem of size 4:
  - M proposal algorithm:
    - All m's get first choice, all w's get last choice
  - W proposal algorithm:
    - All w's get first choice, all m's get last choice

m_1:
m_2:
m_3:
m_4:

w_1:
w_2:
w_3:
w_4:

But there is a stable second choice

- Design a configuration for problem of size 4:
  - M proposal algorithm:
    - All m's get first choice, all w's get last choice
  - W proposal algorithm:
    - All w's get first choice, all m's get last choice
  - There is a stable matching where everyone gets their second choice

m_1:
m_2:
m_3:
m_4:

w_1:
w_2:
w_3:
w_4:

Key ideas

- Formalizing real world problem
  - Model: graph and preference lists
  - Mechanism: stability condition
- Specification of algorithm with a natural operation
  - Proposal
- Establishing termination of process through invariants and progress measure
- Under specification of algorithm
- Establishing uniqueness of solution