CSE 421
Algorithms

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Lecture 30

NP-Completeness

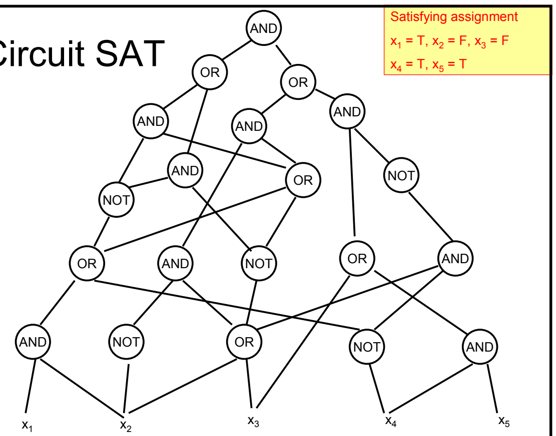
NP-Completeness

- A problem X is NP-complete if
  - X is in NP
  - For every Y in NP, Y $<_P$ X
- X is a "hardest" problem in NP
- To show X is NP complete, we must show how to reduce every problem in NP to X

Cook's Theorem

- The Circuit Satisfiability Problem is NP-Complete
- Circuit Satisfiability
  - Given a boolean circuit, determine if there is an assignment of boolean values to the input to make the output true

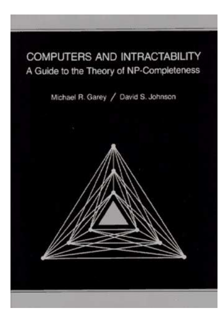
Circuit SAT



Proof of Cook's Theorem

- Reduce an arbitrary problem Y in NP to X
- Let A be a non-deterministic polynomial time algorithm for Y
- Convert A to a circuit, so that Y is a Yes instance iff and only if the circuit is satisfiable

Garey and Johnson



History

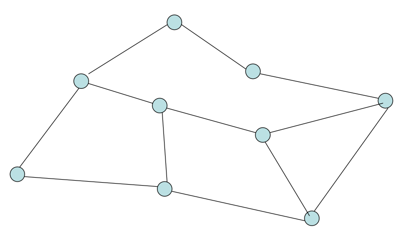
- Jack Edmonds
  - Identified NP
- Steve Cook
  - Cook's Theorem – NP-Completeness
- Dick Karp
  - Identified "standard" collection of NP-Complete Problems
- Leonid Levin
  - Independent discovery of NP-Completeness in USSR

Populating the NP-Completeness Universe

- Circuit Sat $<_P$ 3-SAT
- 3-SAT $<_P$ Independent Set
- Independent Set $<_P$ Vertex Cover
- 3-SAT $<_P$ Hamiltonian Circuit
- Hamiltonian Circuit $<_P$ Traveling Salesman
- 3-SAT $<_P$ Integer Linear Programming
- 3-SAT $<_P$ Graph Coloring
- 3-SAT $<_P$ Subset Sum
- Subset Sum $<_P$ Scheduling with Release times and deadlines

Hamiltonian Circuit Problem

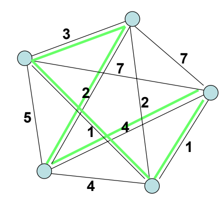
- Hamiltonian Circuit – a simple cycle including all the vertices of the graph



Minimum cost tour highlighted

Traveling Salesman Problem

- Given a complete graph with edge weights, determine the shortest tour that includes all of the vertices (visit each vertex exactly once, and get back to the starting point)



Thm: HC $<_P$ TSP

Number Problems

- Subset sum problem
  - Given natural numbers $w_1, \ldots, w_n$ and a target number W, is their a subset that adds up to exactly W
- Subset sum problem is NP-Complete
- Subset Sum problem can be solved in O(nW) time

Subset sum problem

- The reduction to show Subset Sum is NP-complete involves numbers with n digits
- In that case, the O(nW) algorithm is an exponential time and space algorithm

Course summary
What did we cover in the last 30 lectures?

- Stable Matching
- Models of computation and efficiency
- Basic graph algorithms
  - BFS, Bipartiteness, SCC, Cycles, Topological Sort
- Greedy Algorithms
  - Interval Scheduling, HW Scheduling
  - Correctness proofs
- Dijkstra's Algorithm
- Minimum Spanning Trees
- Recurrences
- Divide and Conquer Algorithms
  - Closest Pair, FFT
- Dynamic Programming
  - Weighted interval scheduling, subset sum, knapsack, longest common subsequence, shortest paths
- Network Flow
  - Ford Fulkerson, Maxflow/mincut, Applications
- NP-Completeness