CSE 421 Algorithms

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Lecture 27

Network Flow Applications

Today's topics

- More network flow reductions
  - Airplane scheduling
  - Image segmentation
  - Baseball elimination

Airplane Scheduling

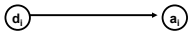
- Given an airline schedule, and starting locations for the planes, is it possible to use a fixed set of planes to satisfy the schedule.
- Schedule
  - [segments] Departure, arrival pairs (cities and times)
- Approach
  - Construct a circulation problem where paths of flow give segments flown by each plane

Compatible segments

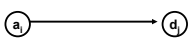
- Segments S_1 and S_2 are compatible if the same plane can be used on S_1 and S_2
  - End of S_1 equals start of S_2, and enough time for turn around between arrival and departure times
  - End of S_1 is different from S_2, but there is enough time to fly between cities

Graph representation

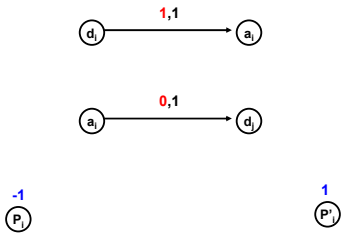
- Each segment, S_i, is represented as a pair of vertices (d_i, a_i, for departure and arrival), with an edge between them.



- Add an edge between a_i and d_j if S_i is compatible with S_j.



Setting up a flow problem



Result

- The planes can satisfy the schedule iff there is a feasible circulation

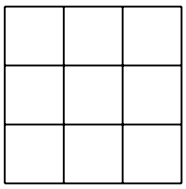
Image Segmentation

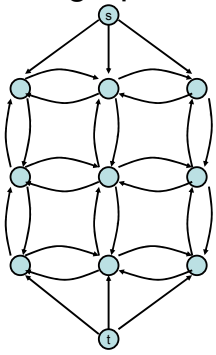
- Separate foreground from background

Image analysis

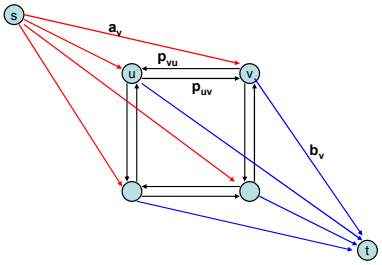
- a_i: value of assigning pixel i to the foreground
- b_i: value of assigning pixel i to the background
- p_{ij}: penalty for assigning i to the foreground, j to the background or vice versa
- A: foreground, B: background
- $Q(A,B) = \Sigma_{\{i \text{ in } A\}} a_i + \Sigma_{\{j \text{ in } B\}} b_j + \Sigma_{\{(i,j) \text{ in } E,\ i \text{ in } A,\ j \text{ in } B\}} p_{ij}$

Pixel graph to flow graph





Mincut Construction



Baseball elimination

- Can the Dung Beetles win the league?
- Remaining games:
  - AB, AC, AD, AD, AD, BC, BC, BC, BD, CD

	W	L
Ants	4	2
Bees	4	2
Cockroaches	3	3
Dung Beetles	1	5

Baseball elimination

- Can the Fruit Flies win the league?
- Remaining games:
  - AC, AD, AD, AD, AF, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, BF, CE, CE, CE, CF, CF, DE, DF, EF, EF

	W	L
Ants	17	12
Bees	16	7
Cockroaches	16	7
Dung Beetles	14	13
Earthworms	14	10
Fruit Flies	12	15

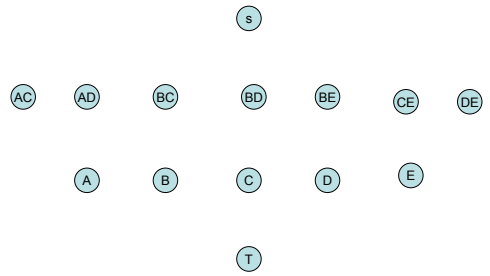
Assume Fruit Flies win remaining games

- Fruit Flies are tied for first place if no team wins more than 19 games
- Allowable wins
  - Ants (2)
  - Bees (3)
  - Cockroaches (3)
  - Dung Beetles (5)
  - Earthworms (5)
- 18 games to play

	W	L
Ants	17	13
Bees	16	8
Cockroaches	16	9
Dung Beetles	14	14
Earthworms	14	12
Fruit Flies	19	15

Remaining games

AC, AD, AD, AD, BC, BC, BC, BC, BC, BD, BE, BE, BE, BE, CE, CE, CE, DE



Network flow applications summary

- Bipartite Matching
- Disjoint Paths
- Airline Scheduling
- Survey Design
- Baseball Elimination
- Project Selection
- Image Segmentation