CSE 421
Algorithms

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Lecture 25

Network Flow Applications

Today's topics

- Problem Reductions
- Circulations
- Lowerbound constraints on flows
- Survey design
- Airplane scheduling

Problem Reduction

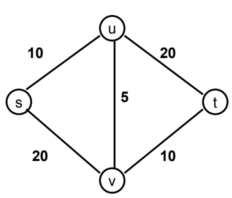
- Reduce Problem A to Problem B
  - Convert an instance of Problem A to an instance Problem B
  - Use a solution of Problem B to get a solution to Problem A
- Practical
  - Use a program for Problem B to solve Problem A
- Theoretical
  - Show that Problem B is at least as hard as Problem A

Problem Reduction Examples

- Reduce the problem of finding the Maximum of a set of integers to finding the Minimum of a set of integers

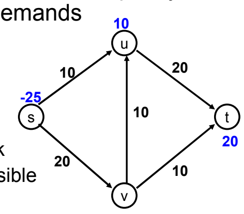
Undirected Network Flow

- Undirected graph with edge capacities
- Flow may go either direction along the edges (subject to the capacity constraints)

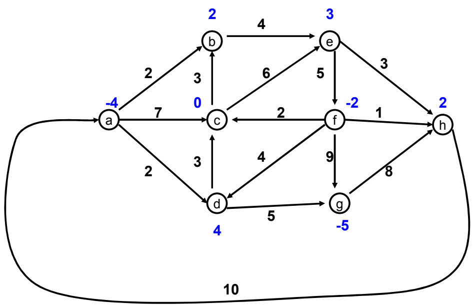


Circulation Problem

- Directed graph with capacities, c(e) on the edges, and demands d(v) on vertices
- Find a flow function that satisfies the capacity constraints and the vertex demands
  - $0 <= f(e) <= c(e)$
  - $f^{in}(v) - f^{out}(v) = d(v)$
- Circulation facts:
  - Feasibility problem
  - d(v) < 0: source; d(v) > 0: sink
  - Must have $\Sigma_v d(v)=0$ to be feasible



Find a circulation in the following graph



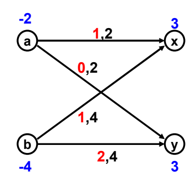
Reducing the circulation problem to Network flow

Formal reduction

- Add source node s, and sink node t
- For each node v, with d(v) < 0, add an edge from s to v with capacity -d(v)
- For each node v, with d(v) > 0, add an edge from v to t with capacity d(v)
- Find a maximum s-t flow. If this flow has size $\Sigma_v$cap(s,v) than the flow gives a circulation satisifying the demands

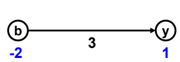
Circulations with lowerbounds on flows on edges

- Each edge has a lowerbound l(e).
  - The flow f must satisfy l(e) <= f(e) <= c(e)



Removing lowerbounds on edges

- Lowerbounds can be shifted to the demands



Formal reduction

- $L_{in}(v)$: sum of lowerbounds on incoming edges
- $L_{out}(v)$: sum of lowerbounds on outgoing edges
- Create new demands d' and capacities c' on vertices and edges
  - $d'(v) = d(v) + l_{out}(v) - l_{in}(v)$
  - $c'(e) = c(e) - l(e)$

Application

- Customized surveys
  - Ask customers about products
    - Only ask customers about products they use
    - Limited number of questions you can ask each customer
    - Need to ask a certain number of customers about each product
    - Information available about which products each customer has used

Details

- Customer $C_1, \ldots, C_n$
- Products $P_1, \ldots, P_m$
- S_i is the set of products used by C_i
- Customer C_i can be asked between c_i and c'_i questions
- Questions about product P_j must be asked on between p_j and p'_j surveys

Circulation construction

Airplane Scheduling

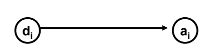
- Given an airline schedule, and starting locations for the planes, is it possible to use a fixed set of planes to satisfy the schedule.
- Schedule
  - [segments] Departure, arrival pairs (cities and times)

Compatible segments

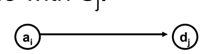
- Segments S_1 and S_2 are compatible if the same plane can be used on S_1 and S_2
  - End of S_1 equals start of S_2, and enough time for turn around between arrival and departure times
  - End of S_1 is different from S_2, but there is enough time to fly between cities

Graph representation

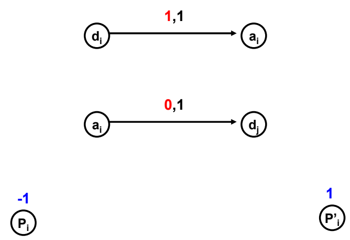
- Each segment, S_i, is represented as a pair of vertices (d_i, a_i, for departure and arrival), with an edge between them.



- Add an edge between a_i and d_j if S_i is compatible with S_j.



Setting up a flow problem



Result

- The planes can satisfy the schedule iff there is a feasible circulation