CSE 421
Algorithms

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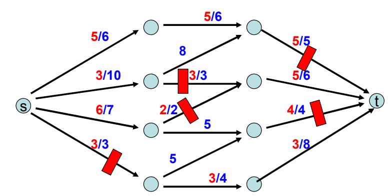
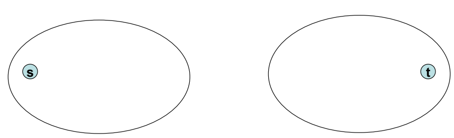
Lecture 24
Maxflow MinCut Theorem

Find a minimum value cut

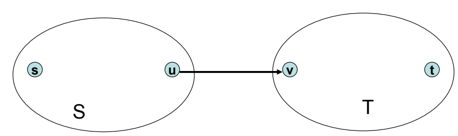


MaxFlow – MinCut Theorem

- There exists a flow which has the same value of the minimum cut
- Proof: Consider a flow where the residual graph has no s-t path with positive capacity
- Let S be the set of vertices in G_R reachable from s with paths of positive capacity



Let S be the set of vertices in G_R reachable from s with paths of positive capacity



What is Cap(u,v) in G_R?

What can you say about Cap(u,v) and Flow(u,v) in G?

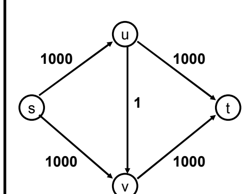
What can you say about Cap(v,u) and Flow(v,u) in G?

Max Flow - Min Cut Theorem

- Ford-Fulkerson algorithm finds a flow where the residual graph is disconnected, hence FF finds a maximum flow.
- If we want to find a minimum cut, we begin by looking for a maximum flow.

Performance

- The worst case performance of the Ford-Fulkerson algorithm is horrible



Better methods of finding augmenting paths

- Find the maximum capacity augmenting path
  - $O(m^2 \log(C))$ time
- Find the shortest augmenting path
  - $O(m^2 n)$
- Find a blocking flow in the residual graph
  - $O(mn \log n)$

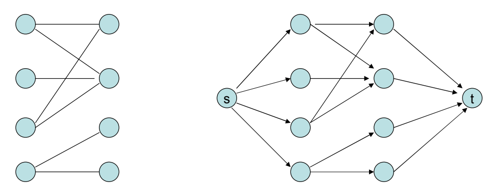
Bipartite Matching

- A graph G=(V,E) is bipartite if the vertices can be partitioned into disjoints sets X,Y
- A matching M is a subset of the edges that does not share any vertices
- Find a matching as large as possible

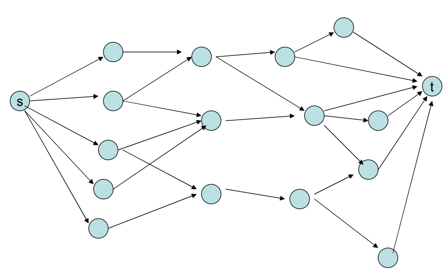
Application

- A collection of teachers
- A collection of courses
- And a graph showing which teachers can teach which course

Converting Matching to Network Flow



Finding edge disjoint paths



Theorem

- The maximum number of edge disjoint paths equals the minimum number of edges whose removal separates s from t