CSE 421
Algorithms

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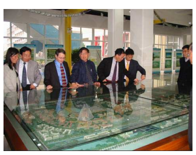
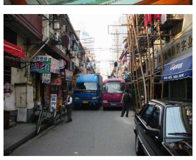
Lecture 18
Dynamic Programming








Announcements

- Wednesday class will meet in CSE 305.

Dynamic Programming

- The most important algorithmic technique covered in CSE 421
- Key ideas
  - Express solution in terms of a polynomial number of sub problems
  - Order sub problems to avoid recomputation

Today - Examples

- Examples
  - Optimal Billboard Placement
    - Text, Solved Exercise, Pg 307
  - Linebreaking with hyphenation
    - Compare with HW problem 6, Pg 317
  - String concatenation
    - Text, Solved Exercise, Page 309

Billboard Placement

- Maximize income in placing billboards
  - (p_i, v_i), v_i: value of placing billboard at position p_i
- Constraint:
  - At most one billboard every five miles
- Example
  - {(6,5), (7,6), (12, 5), (14, 1)}

Opt[k]

- What are the sub problems?

Opt[k] = fun(Opt[0],…,Opt[k-1])

- How is the solution determined from sub problems?

Solution

```
j = 0;          // j is five miles behind the current value
                // the last valid location for a billboard, if one placed at P[k]
for k := 1 to n
      while (P[j] < P[k] – 5)
            j := j + 1;
      j := j – 1;
      Opt[k] = Max(Opt[k-1], V[k] + Opt[j]);
```

Optimal line breaking and hyphen-ation

- Problem: break lines and insert hyphens to make lines as balanced as possible
- Typographical considerations:
  - Avoid excessive white space
  - Limit number of hyphens
  - Avoid widows and orphans
  - Etc.

Penalty Function

- Pen(i, j) – penalty of starting a line a position i, and ending at position j

Opt-i-mal line break-ing and hyph-en-a-tion is com-put-ed with dy-nam-ic pro-gram-ming

- Key technical idea
  - Number the breaks between words/syllables

Opt[k]

- What are the sub problems?

Opt[k] = fun(Opt[0],…,Opt[k-1])

- How is the solution determined from sub problems?

Solution

```
for k := 1 to n
        Opt[k] := infinity;
        for j := 1 to n-1
                Opt[k] := Min(Opt[k], Opt[j] + Pen(j, k));
```

But what if you want to layout the text?

- And not just know the minimum penalty?

Solution

```
for k := 1 to n
        Opt[k] := infinity;
        for j := 1 to n-1
                temp := Opt[j] + Pen(j, k);
                if (temp < Opt[k])
                        Opt[k] = temp;
                        Best[k] := j;
```

String approximation

- Given a string S, and a library of strings B = {b_1, …b_k}, construct an approximation of the string S by using copies of strings in B.

B = {abab, bbbaaa, ccbb, ccaacc}

S = abaccbbbaabbccbbccaabab

Formal Model

- Strings from B assigned to non-overlapping positions of s
- Strings from B may be used multiple times
- Cost of δ for unmatched character in s
- Cost of γ for mismatched character in s
  - MisMatch(i, j) – number of mismatched characters of b_j, when aligned starting with position i in s.

Opt[k]

- What are the sub problems?

Opt[k] = fun(Opt[0],…,Opt[k-1])

- How is the solution determined from sub problems?

Solution

```
for i := 1 to n
        Opt[k] = Opt[k-1] + δ;
        for j := 1 to |B|
                p = i – len(b_j);
                Opt[k] = min(Opt[k],  Opt[p-1] + γ MisMatch(p, j));
```