CSE 421
Algorithms

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Lecture 8
Greedy Algorithms: Homework Scheduling and Optimal Caching

Announcements

- Monday, October 17
  - Class will meeting in CSE 305
  - Tablets again!
  - Read sections 4.4 and 4.5 before class
  - Lecture will be designed with the assumption that you have read the text

Greedy Algorithms

- Solve problems with the simplest possible algorithm
- The hard part: showing that something simple actually works
- Today's problem
  - Homework Scheduling
  - Optimal Caching

Homework Scheduling

- Tasks to perform
- Deadlines on the tasks
- Freedom to schedule tasks in any order

Scheduling tasks

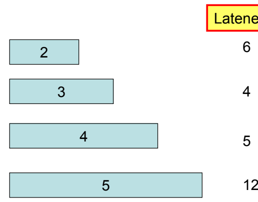
- Each task has a length t_i and a deadline d_i
- All tasks are available at the start
- One task may be worked on at a time
- All tasks must be completed

- Goal minimize maximum lateness
  - Lateness = $f_i - d_i$ if $f_i >= d_i$

Example

Show the schedule 2, 3, 4, 5 first and compute lateness

Lateness



Greedy Algorithm

- Earliest deadline first
- Order jobs by deadline

- This algorithm is optimal

This result may be surprising, since it ignores the job lengths

Analysis

- Suppose the jobs are ordered by deadlines, $d_1 <= d_2 <= . . . <= d_n$
- A schedule has an *inversion* if job j is scheduled before i where j > i

- The schedule A computed by the greedy algorithm has no inversions.
- Let O be the optimal schedule, we want to show that A has the same maximum lateness as O

Proof

- Lemma: There is an optimal schedule with no idle time.

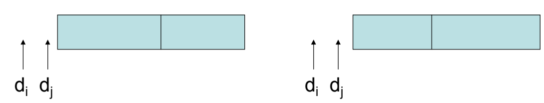
- Lemma: There is an optimal schedule with no inversions and no idle time.

- Let O be an optimal schedule k inversions, we construct a new optimal schedule with k-1 inversions

If there is an inversion, there is an inversion of adjacent jobs

Interchange argument

- Suppose there is a pair of jobs i and j, with i < j, and j scheduled immediately before i. Interchanging i and j does not increase the maximum lateness. Recall, $d_i <= d_j$



Result

- Earliest Deadline First algorithm constructs a schedule that minimizes the maximum lateness

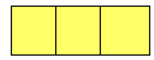
Extensions

- What if the objective is to minimize the sum of the lateness?
  - EDF does not seem to work
- If the tasks have release times and deadlines, and are non-preemptable, the problem is NP-complete
- What about the case with release times and deadlines where tasks are preemptable?

Optimal Caching

- Caching problem:
  - Maintain collection of items in local memory
  - Minimize number of items fetched

Caching example



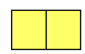
A, B, C, D, A, E, B, A, D, A, C, B, D, A

Optimal Caching

- If you know the sequence of requests, what is the optimal replacement pattern?
- Note – it is rare to know what the requests are in advance – but we still might want to do this:
  - Some specific applications, the sequence is known
  - Competitive analysis, compare performance on an online algorithm with an optimal offline algorithm

Farthest in the future algorithm

- Discard element used farthest in the future



A, B, C, A, C, D, C, B, C, A, D

Correctness Proof

- Sketch
- Start with Optimal Solution O
- Convert to Farthest in the Future Solution F-F
- Look at the first place where they differ
- Convert O to evict F-F element
  - There are some technicalities here to ensure the caches have the same configuration . . .