CSE 421
Algorithms

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Lecture 6

Graph Theory

Draw a picture of David Notkin

To submit your drawing, press the button

Describe an algorithm to determine if an undirected graph has a cycle

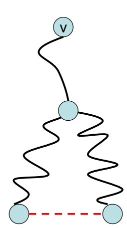
Cycle finding

- Does a graph have a cycle?
- Find a cycle
- Find a cycle through a specific vertex v

- Linear runtime: O(n+m)

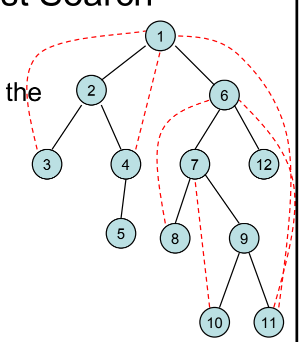
Find a cycle through a vertex v

- Not obvious how to do this with BFS from vertex v



Depth First Search

- Each edge goes between vertices on the same branch
- No cross edges

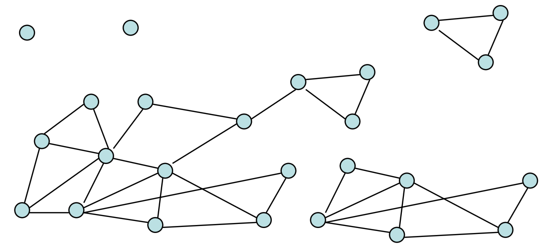


A DFS from vertex v gives a simple algorithm for finding a cycle containing v

How does this algorithm work and why?

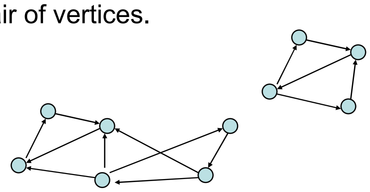
Connected Components

- Undirected Graphs

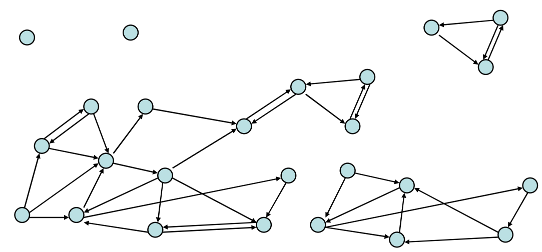


Directed Graphs

- A Strongly Connected Component is a subset of the vertices with paths between every pair of vertices.

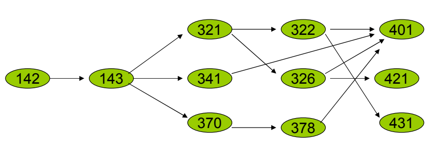


Identify the Strongly Connected Components

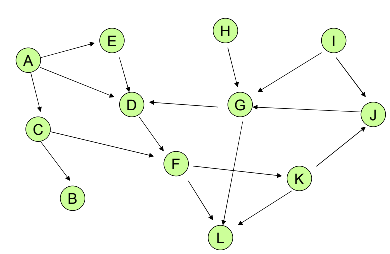


Topological Sort

- Given a set of tasks with precedence constraints, find a linear order of the tasks

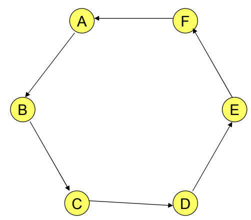


Find a topological order for the following graph



If a graph has a cycle, there is no topological sort

- Consider the first vertex on the cycle in the topological sort
- It must have an incoming edge

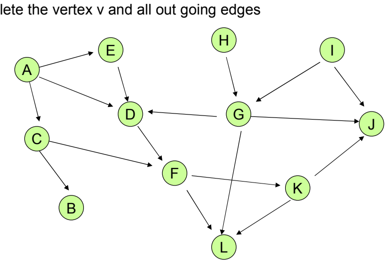


Lemma: If a graph is acyclic, it has a vertex with in degree 0

- Proof:
  – Pick a vertex v_1, if it has in-degree 0 then done
  – If not, let (v_2, v_1) be an edge, if v_2 has in-degree 0 then done
  – If not, let (v_3, v_2) be an edge . . .
  – If this process continues for more than n steps, we have a repeated vertex, so we have a cycle

Topological Sort Algorithm

While there exists a vertex v with in-degree 0
  Output vertex v
  Delete the vertex v and all out going edges



Details for O(n+m) implementation

- Maintain a list of vertices of in-degree 0
- Each vertex keeps track of its in-degree
- Update in-degrees and list when edges are removed
- m edge removals at O(1) cost each