CSE 421
Algorithms

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Lecture 3

Classroom Presenter Project

- Understand how to use Pen Computing to support classroom instruction
- Writing on electronic slides
- Distributed presentation
- Student submissions
- Classroom Presenter 2.0, started January 2002
  - www.cs.washington.edu/education/dl/presenter/
- Classroom Presenter 3.0, started June 2005

Key ideas for Stable Matching

- Formalizing real world problem
  - Model: graph and preference lists
  - Mechanism: stability condition
- Specification of algorithm with a natural operation
  - Proposal
- Establishing termination of process through invariants and progress measure
- Underspecification of algorithm
- Establishing uniqueness of solution

Question

- Goodness of a stable matching:
  - Add up the ranks of all the matched pairs
  - M-rank, W-rank
- Suppose that the preferences are completely random
  - If there are n M's, and n W's, what is the expected value of the M-rank and the W-rank

What is the run time of the Stable Matching Algorithm?

Initially all m in M and w in W are free
While there is a free m **Executed at most n^2 times**
    w highest on m's list that m has not proposed to
    if w is free, then match (m, w)
    else
        suppose (m_2, w) is matched
        if w prefers m to m_2
            unmatch (m_2, w)
            match (m, w)

O(1) time per iteration

- Find free m
- Find next available w
- If w is matched, determine m_2
- Test if w prefer m to m_2
- Update matching

What does it mean for an algorithm to be efficient?

Definitions of efficiency

- Fast in practice
- Qualitatively better worst case performance than a brute force algorithm

Polynomial time efficiency

- An algorithm is efficient if it has a polynomial run time
- Run time as a function of problem size
  - Run time: count number of instructions executed on an underlying model of computation
  - T(n): maximum run time for all problems of size at most n

Polynomial Time

- Algorithms with polynomial run time have the property that increasing the problem size by a constant factor increases the run time by at most a constant factor (depending on the algorithm)

Why Polynomial Time?

- Generally, polynomial time seems to capture the algorithms which are efficient in practice
- The class of polynomial time algorithms has many good, mathematical properties

Ignoring constant factors

- Express run time as O(f(n))
- Emphasize algorithms with slower growth rates
- Fundamental idea in the study of algorithms
- Basis of Tarjan/Hopcroft Turing Award

Why ignore constant factors?

- Constant factors are arbitrary
  - Depend on the implementation
  - Depend on the details of the model
- Determining the constant factors is tedious and provides little insight

Why emphasize growth rates?

- The algorithm with the lower growth rate will be faster for all but a finite number of cases
- Performance is most important for larger problem size
- As memory prices continue to fall, bigger problem sizes become feasible
- Improving growth rate often requires new techniques

Formalizing growth rates

- T(n) is O(f(n)) $[T : Z^+ \rightarrow R^+]$
  - If sufficiently large n, T(n) is bounded by a constant multiple of f(n)
  - Exist c, n_0, such that for $n > n_0$, $T(n) < c\, f(n)$
- T(n) is O(f(n)) will be written as: T(n) = O(f(n))
  - Be careful with this notation

Prove $3n^2 + 5n + 20$ is $O(n^2)$

Lower bounds

- T(n) is Ω(f(n))
  - T(n) is at least a constant multiple of f(n)
  - There exists an n_0, and $\varepsilon > 0$ such that $T(n) > \varepsilon f(n)$ for all $n > n_0$
- Warning: definitions of Ω vary
- T(n) is Θ(f(n)) if T(n) is O(f(n)) and T(n) is Ω(f(n))

Useful Theorems

- If lim (f(n) / g(n)) = c for c > 0 then f(n) = Θ(g(n))
- If f(n) is O(g(n)) and g(n) is O(h(n)) then f(n) is O(h(n)))
- If f(n) is O(h(n)) and g(n) is O(h(n)) then f(n) + g(n) is O(h(n))

Ordering growth rates

- For $b > 1$ and $x > 0$
  - $\log_b n$ is $O(n^x)$

- For $r > 1$ and $d > 0$
  - n^d is $O(r^n)$