CSci 421
Introduction to Algorithms

Homework Assignment 3
Due: Wednesday, 28 Jan 2004

Winter 2004 Handout 3

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Reading Assignment:

Read Chapter 5, 6.6, 6.8, 6.11.1.

Homework:

1. Simulate the Maximum Consecutive Subsequence algorithm on the following sequence.

   1, 2, -2, 1, 1, 2, -6, 1, 3, 10

   Show the values of `Suffix_Max` and `Global_Max` after each iteration through the main loop, as well as showing the starting and ending indices of the subsequences to which they implicitly refer.

2. Run the Knapsack algorithm (Fig 5.10, pg 110) on the sequence of weights $k_1 = 5$, $k_2 = 2$, $k_3 = 4$, $k_4 = 3$, and $k_5 = 6$], with knapsack capacity $K = 16$. Show a table like Fig 5.11 to summarize the computation.

3. Give an algorithm to solve the following variant of the Knapsack Problem: In addition to the knapsack capacity K and the weights (also called sizes) k_i of the n objects that may be placed in the knapsack, suppose you are also given *values* for each, i.e., n positive real numbers $v_i, 1 \leq i \leq n$, and the goal is select a subset of the items so that (1) their total weight does not exceed the knapsack capacity, and (2) their total value is as large as possible subject to (1). I.e., find a set $I \subseteq \{1, 2, \ldots, n\}$ such that $\sum_{i \in I} v_i$ is maximized subject to the constraint that $\sum_{i \in I} k_i \leq K$. (Note $\leq K$ rather than $= K$.) Briefly explain why your algorithm is correct. In particular, state carefully the "induction hypothesis" that characterizes it. Analyze its running time.

4. Given two sorted lists of numbers $x_1 < x_2 < \cdots < x_n$ and $y_1 < y_2 < \cdots < y_m$, and a number Z, give an algorithm to find the set

   $$\{(i, j) \mid 1 \leq i \leq n; 1 \leq j \leq m \text{ such that , } x_i + y_j = Z\}.$$

   Analyze its running time. Time $O(n + m)$ is possible.

5. 6.64.