Name: ______________________

CSE 421: Introduction to Algorithms

"Sample" Final Exam

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Take "Sample Final" with a big grain of salt. The questions below, although they are ones I have used before, are not all from the same year, don't reflect the expected time limit of a real test (e.g., may be way too long/hard...), and are not necessarily representative of questions I am likely to use, but do cover appropriate material.

1. Depth first search of a directed graph divides the edges into four categories. Name them, explain how the depth first search algorithm would distinguish the four cases, give one example graph where all four kinds of edge arise, and explain why no other kinds are possible in general.

2. Define the following:

   (a) P:

   (b) NP:

   (c) A is polynomial time reducible to B ($A \leq^p B$):

   (d) A is NP-complete:

   What is the practical significance of the statement "A is NP-complete"?

3. Circle the most precise classification applicable to each of the following pairs of functions:

   A $f(n) = O(g(n))$ but it is not true that $f(n) = \Omega(g(n))$.

   B $f(n) = \Theta(g(n))$.

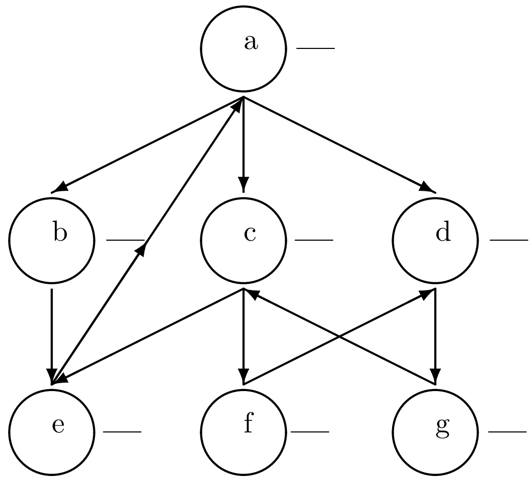
   C $f(n) = o(g(n))$.

   D $f(n) = \omega(g(n))$.

	$f(n)$	$g(n)$					
a	$500n^3 - 1729n^2 + n - 7$	n^3	....	A	B	C	D
b	$500n^3 - 1729n^2 + n - 7$	n^4	....	A	B	C	D
c	$n/\log n$	n	....	A	B	C	D
d	$n^2(\log n)^5$	$n^{2.1}$	....	A	B	C	D
e	n^{1000}	$n!$	....	A	B	C	D
f	n^{1000}	1.01^n	....	A	B	C	D
g	$\sum_{i=1}^{n}(i^3 + 5i^2)$	n^4	....	A	B	C	D
g	$\sum_{i=1}^{n}(i^3 + 5i^2)$	n^3	....	A	B	C	D

4. Do a depth-first search of the following directed graph, starting from vertex a. Assign (increasing) depth-first numbers to the vertices (write them in the blanks next to the vertices on the figure), and classify the edges as tree-, forward-, back, or cross-edges (circle the correct choice in the table.

   Just to make the solution unique, so as to save wear-and-tear on the poor, pitiful grader (me), at any point where your algorithm has a choice as to which edge to explore next, assume it chooses the edge leading to the vertex having the *alphabetically first* label among the candidate vertices. E.g., given unexplored edges to c and d, and an explored edge to b, choose the edge to c.



Edge	Type (Circle correct choice)
a→b	Tree- Forward- Cross- Back-
a→c	Tree- Forward- Cross- Back-
a→d	Tree- Forward- Cross- Back-
b→e	Tree- Forward- Cross- Back-
c→e	Tree- Forward- Cross- Back-
c→f	Tree- Forward- Cross- Back-
d→g	Tree- Forward- Cross- Back-
e→a	Tree- Forward- Cross- Back-
f→d	Tree- Forward- Cross- Back-
g→c	Tree- Forward- Cross- Back-

5. (20 points) In this problem you are to find the optimal parenthesization of a matrix-chain product $A_1 A_2 A_3 A_4$, where

$$\begin{aligned} A_1 &\text{ is } 4 \times 5, \\ A_2 &\text{ is } 5 \times 4, \\ A_3 &\text{ is } 4 \times 2, \text{ and} \\ A_4 &\text{ is } 2 \times 3. \end{aligned}$$

(a) We have filled in all of your table except the $m[1, 4]$ entry. Fill in that one. Show your work.

	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$i = 1$	0	80	80	
$i = 2$		0	40	70
$i = 3$			0	24
$i = 4$				0

(b) Show the optimal parenthesization of $A_1 A_2 A_3 A_4$, and explain how you derived it from the table in part (a).

6. (40 points) Suppose you are given an edge-weighted undirected graph $G = (V, E)$, together with a minimum spanning tree T of G. Now suppose that one particular edge $e = (u, v)$ in G is given a new weight w', which is not equal to its weight w in G. T may or may not be a minimum spanning tree of this modified graph. Give an algorithm to decide whether T is a minimum spanning tree of the modified graph, and if not, to construct a new minimum spanning tree that differs from T by the deletion of one edge and the addition of one edge. There are 4 relevant cases, depending on whether $w' < w$ or not, and whether e is an edge in T or not. Two of these cases are easy, requiring only $O(1)$ steps. One case will probably need $O(|V|)$ steps (even if $|V| \ll |E|$). The fourth case will probably require $O(|E|)$ steps. Describe the 4 cases separately, briefly arguing correctness and sketching a complexity analysis for each.
[Circle Yes/No appropriately for the 4 cases below.]

(a) $w' < w$? Yes No e in T? Yes No

(b) $w' < w$? Yes No e in T? Yes No

(c) $w' < w$? Yes No e in T? Yes No

(d) $w' < w$? Yes No e in T? Yes No