CSci 421
Introduction to Algorithms

Homework Assignment 5
Due: Friday, 23 Feb 2001

Winter 2001 February 19, 2001

Reading in Chapter 7: 7.1, 7.3, 7.4, 7.5, 7.6 (although I gave a different algorithm in lecture), 7.7, 7.8, 7.9 intro + 7.9.2, 7.10, 7.11, 7.13.

Homework:

1. Let G denote a connected undirected graph with weighted edges. The weights need not be distinct. A *cut* of G is simply a partition of its vertices into two nonempty subsets; an edge *crosses the cut* if one end point is in one part of the partition and the other endpoint is in the other part.

   Prove or disprove the following:
   Let e be an edge of G. Then the following statements are equivalent:

   (a) There exists a minimum spanning tree containing the edge e;

   (b) Every simple cycle containing e contains another edge at least as heavy as e;

   (c) There exists a cut crossed by e for which every edge crossing the cut is at least as heavy as e;

2. 7.7.

3. Let $G = (V, E)$ be a connected undirected graph, and v a designated vertex of G. A depth first search of G starting at v may produce different results depending on the order in which edges are examined at each vertex. In particular, the set $T \subseteq E$ of edges categorized as "tree edges" by DFS (defining a spanning tree of G) may depend on the edge order.

   (a) Give an example of this, on a graph with $|V| \leq 5$.

   (b) We say a spanning tree T (with root v) "is a DFS tree for v" if there is an ordering of the edges of E incident to each vertex such that T is the tree constructed by DFS(v).
   Disprove: Every spanning tree is a DFS tree.

   (c) Give a necessary and sufficient condition for a spanning tree T rooted at v to be a DFS tree for v.

   (d) Give an efficient algorithm ($O(|E| + |V|)$ if possible) to determine, given G, v and a spanning tree T, whether T is a DFS tree for v.

4. 7.16. You may use *in*creasing DFS numbers and LOW values (as in lecture) rather than decreasing numbers/HIGH values (as in text) if you prefer. Say which you're doing.

5. 7.17. Is the same true if all cross edges are also removed? Prove it.

6. 7.38.

7. 7.91. Assume every vertex in G is incident to at least one edge.